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FINAL REPORT ON THE SPECIAL YEAR IN NUMERICAL ANALYSIS, 1980-81--ETC(U)
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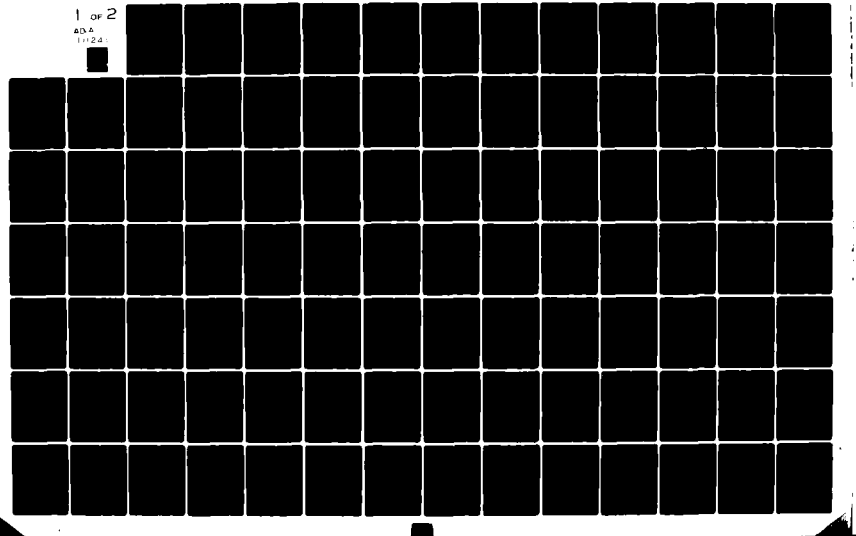

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18 SUPPLEMENTARY NOTES

19 KEY WORDS (Continue on reverse side if necessary and identify by block number)

20 ABSTRACT (Continue on reverse side if necessary and identify by block number)

[illegible] the Department of Mathematics at the University of Maryland had a [illegible] in some branch of mathematics. This past year the topic was [illegible] Analysis. The Special Year in 1980-81 was jointly sponsored by the Air Force Office of Scientific Research and the Mathematics Department. The [illegible] to advance the state of the art in numerical analysis by bringing [illegible] the leading experts in the field for formal lectures and informal [illegible] recent progress, current problems, and future trends. (CONTINUED)

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ITEM #20, CONTINUED: Special emphasis was placed on numerical [illegible] differential equations, global continuation methods, numerical [illegible] statistics, numerical linear algebra, and numerical problems connected with [illegible] functions. The activities in statistics, numerical linear algebra, and [illegible] problems took place in the Fall Semester and those in the areas of [illegible] partial differential equations took place in the Spring. In the areas of [illegible] continuation methods, the activities were spread throughout the year. The direction of this report is devoted to each of these major areas.

The lectures and the extensive opportunities for informal discussion, [illegible] year provided an excellent opportunity for exchange of information and ideas between the members of the large and active numerical analysis group at the University and the visiting mathematicians. We believe the benefits [illegible] both to the University and to the international numerical analysis community.

FINAL REPORT ON THE SPECIAL YEAR IN NUMERICAL ANALYSIS, 1980-81

AFOSR-80-0251

1. Introduction

Each year the Department of Mathematics at the University of Maryland has a Special Year in some branch of mathematics. This past year the topic was Numerical Analysis. The Special Year in 1980-81 was jointly sponsored by the Air Force Office of Scientific Research and the Mathematics Department. The main goal of the year was to advance the state of the art in numerical analysis by bringing together the leading experts in the field for formal lectures and informal discussions of recent progress, current problems, and future trends. We placed special emphasis on numerical solution of partial differential equations, global continuation methods, numerical methods in statistics, numerical linear algebra, and numerical problems connected with special functions. The activities in statistics, numerical linear algebra, and special functions took place in the Fall Semester and those in the numerical solution of partial differential equations took place in the Spring. In the area of global continuation methods, the activities were spread throughout the year. A subsection of this report is devoted to each of these major activities.

Through the lectures and the extensive opportunities for informal discussions, the Special Year provided an excellent opportunity for exchange of information and ideas between the members of the large and active numerical analysis group at the University and the visiting mathematicians. We believe the benefits will be substantial both to the University and to the international numerical analysis community.

2. Activities

a. Numerical Solution of Partial Differential Equations

A wide variety of important problems in science and engineering are formulated as initial-boundary value problems for partial differential equations, the numerical solution of which is one of the most important areas of numerical analysis. In order to survey the central problems and trends in this area, we invited a total of 30 distinguished visitors to the campus during the Spring Semester. These 30 visitors represented nearly all important subfields of the area. The lectures by the visitors were attended by Maryland students and faculty who are working in the area or have an interest in it, members of the numerical analysis community of the Washington area (e.g., from the Naval Surface Weapons Center), and of course the other visitors in residence at the time. In addition to the lectures there was ample opportunity for informal discussions. These discussions were especially fruitful and a number of joint research projects have grown out of them. Many of the visitors submitted written versions of their lectures. These range from extended abstracts, to systematic survey papers, to standard research papers. This collection of papers has been published as part of the Lecture Note Series of the Mathematics Department. The activities in this area were loosely divided between finite element and finite difference methods. The program in Numerical Solution of Partial Differential Equations was directed by Professors I. Babuška, T.-P. Liu, and J. Osborn.

See Attachment A for a list of participants and lectures.

b. Numerical Methods in Statistics, Numerical Linear Algebra, and Computation of Special Functions

The fields of statistics, linear algebra, and special functions are usually regarded as distinct by the mathematical community. In applications, however, there is considerable overlap: each field leads to numerical and other problems in at least one of the other two. The increasing speed and capacity of modern computers is bringing more and more of these problems into the realm of feasibility.

In order to generate as many contacts as possible between research workers in the three fields, it was decided to concentrate activities into a single conference. This was held from October 2 to October 8, 1980 at the Adult Education Center on the College Park campus. The conference was advertised in the Notices of the American Mathematical Society, SIAM News, Bulletin of the Institute of Mathematical Statistics, AMSTAT News, and the Washington Statistical Society. In addition, announcements of the conference were sent directly by mail to a list of approximately 1650 individuals, including all members of S.I.A.M. resident in North America. The organizing committee consisted of F. W. J. Olver, G. Stewart and G. Yang.

In all, 132 individuals registered for and attended the conference. This included 67 from the University of Maryland, 29 from other universities, 18 from government agencies, and 14 from industry. (Four gave no affiliation.) The attendees from the University of Maryland represented 11 different departments or programs.

The program consisted of 10 invited 1-hour lectures and 21 30-minute lectures. Speakers in the second category were selected by the organizing

committee from extended abstracts submitted in advance. There were also sessions for 15-minute contributed papers.

See Attachment B for the conference program, abstracts of the 1-hour invited lectures, abstracts of the 30-minute selected talks, and a list of registrants.

c. Homotopy Continuation Methods

Homotopy continuation methods are directed towards solving systems of equations in situations where approximate solutions are not available and quasi-newton type methods fail. Smooth continuation methods were emphasized, although simplicial methods were represented by Eaves and Peitgen. Keller's and Watson's lectures paid special attention to applications to physical problems. Harrison and Smale gave lectures aimed at topics relevant to theorical understanding of families of periodic orbits (Harrison) and the number of steps needed to implement methods (Smale). Interactions with visitors have led to two papers being written. The homotopy program was organized by Professor J. A. Yorke.

See Attachment C for a list of participants and lectures.

LIST OF ATTACHMENTS

ATTACHMENT A: List of Participants and Lectures in Numerical PDE Portion of Special Year.

ATTACHMENT B: Program, Abstracts of the 1-Hour Invited Lectures, Abstracts of the 30-Minute Selected Talks, and List of Registrants for the Conference on Applications of Numerical Analysis and Special Functions in Statistics.

ATTACHMENT C: List of Participants and Lectures in Global Continuation Methods Portion of Special Year.

ATTACHMENT A: List of Participants and Lectures in Numerical PDE Portion of Special Year

List of Participants

PROFESSOR GARTH BAKER
Harvard University

PROFESSOR GARRETT BIRKHOFF
Harvard University

PROFESSOR JAMES BRAMBLE
Cornell University

PROFESSOR F. BREZZI
Universite di Pavia
ITALY

PROFESSOR P.G. CIARLET
Universite Pierre et Marie
Curie, FRANCE

PROFESSOR JIM DOUGLIS, JR.
University of Chicago

PROFESSOR T. DUPONT
University of Chicago

PROFESSOR B. ENQUIST
University of California,
Los Angeles

DR. RICHARD EWING
Mobil Field Research
Laboratory

PROFESSOR R. FALK
Rutgers University

PROFESSOR P. GARABEDIAN
Courant Institute

PROFESSOR J. GLIMM
Rockefeller University

PROFESSOR AMIRAM HARTEN
Tel-Aviv University
ISRAEL

PROFESSOR LING HSAIO
Brown University and
Academia Sinica, Peking
PEOPLES REPUBLIC OF CHINA

PROFESSOR P. LAX
Courant Institute

PROFESSOR MITCHELL LUSKIN
Courant Institute

PROFESSOR A. MAJADA
University of California,
Berkeley

PROFESSOR J. NITSCHE
Inst. Fur Angewandte Math.
GERMANY

PROFESSOR J.T. ODEN
University of Texas,
Austin

PROFESSOR J. OLIGER
Stanford University

PROFESSOR A. SCHATZ
Cornell University

PROFESSOR RIDGWAY SCOTT
University of Wisconsin
Mathematics Research Center

PROFESSOR G. STRANG
Massachusetts Institute
of Technology

PROFESSOR ROGER TEMAM
Universite de Paris
FRANCE

PROFESSOR VIDAR THOMEE
Chalmers University of
Technology

PROFESSOR LARS WAHLBIN
Cornell University

PROFESSOR W. WENDLAND
Technische Hochschule
Darmstadt, GERMANY

PROFESSOR B. WENDROFF
Los Alamos Scientific
Laboratory

PROFESSOR MARY WHEELER
Rice University

PROFESSOR MILOS ZLAMAL
Technical University
CZECHLOSLOVAKIA

List of Lectures

DATE	SPEAKER	TOPIC
Jan 16	V. Thomee	Single step methods for linear differential equations in Banach spaces, PART I
Jan 20	V. Thomee	Single step methods for linear differential equations in Banach spaces, PART II
Jan 20	J. Bramble	Remarks on Lagrange multiplier techniques in conjunction with finite element approximations in various elliptic problems
Jan 22	G. Birkhoff	Adapting Courant-Friedrichs Levy to the 1980's
Jan 27	V. Thomee	Single step methods for linear differential equations in Banach spaces, PART III
Jan 29	R. Scott	A comparison of laboratory experiments with a model equation for water waves
Feb 5	M. Luskin	Analysis of a fractional step method for fluid flow in a pipe
Feb 10	A. Harten	On random choice methods for hyperbolic conservation laws
Feb 10	M. Luskin	On a finite element method to solve the critcality eigenvalue problem for the transport equation
Feb 10	P. Lax	Convergence almost everywhere of random choice schemes
Feb 12	R. Temam	Variational problems in mechanics (plasticity) PART I
Feb 12	P. Ciarlet	Questions of existence in non linear elasticity
Feb 17	P. Ciarlet	Justification of the von Karman equations
Feb 17	M. Wheeler	Mixed methods for miscible displacement problems
Feb 18	R. Temam	Variational problems in mechanics (plasticity) PART II
Feb 19	J. Nitsche	The method of straightening the free boundary in moving boundary problems
Feb 24	J. Douglas	Numerical simulation of flow in porus media
Feb 24	J. Oden	Analysis of some contact problems in nonlinear elasticity
Feb 26	G. Baker	Spectral approximation in Riemannian geometry

List of Lectures (continued)

DATE	SPEAKER	TOPIC
Feb 26	R. Ewing	Alternating directional multistep procedure for nonlinear parabolic P.D.E.'s
Feb 27	J. Nitsche	A remarkable approximation property of finite elements and its consequences
Mar 3	B. Enquist	Radiation boundary conditions at computational boundaries
Mar 3	B. Wendroff	Defect corrections, multigrids and selected applications, PART I
Mar 5	L. Wahlbin	On maximum norm estimates in finite element methods PART I
Mar 6	W. Wendland	Asymptotic convergence of boundary element methods
Mar 10	B. Enquist	Flux splittings in compressible flow computations
Mar 10	L. Wahlbin	On maximum norm estimates in finite element methods PART II
Mar 12	B. Wendroff	Defect corrections, multigrids and selected applications, PART II
Mar 12	W. Wendland	Integral equation methods for mixed boundary value problems
Mar 12	L. Wahlbin	On maximum norm estimates in finite element methods PART III
Mar 24	A. Majda	Vortex methods in fluid flow
Mar 24	T. Dupont	Mesh modification in finite element methods
Mar 26	J. Glimm	Hydrodynamics without diffusion: Theory, computation and application, PART I
Mar 31	A. Schatz	Singular functions in the finite element method
Apr 1	A. Majda	A theory for Mach Stern formation in reacting shock fronts
Apr 2	F. Brezzi	Finite dimensional approximation of nonlinear problems, PART I
Apr 7	A. Schatz	Boundedness in L_∞ of the Ritz projection
Apr 7	M. Zlamal	Galerkin-finite element methods for the solution of nonlinear evolution equations, PART I

List of Lectures (continued)

DATE	SPEAKER	TOPIC
Apr 8	J. Glimm	Hydrodynamics without diffusion: Theory, computation and application, PART II
Apr 9	L. Hsiao	Overtaking of shock waves in steady two dimensional supersonic flows
Apr 9	J. Oliger	Adaptive difference methods for time dependent problems
Apr 10	J. Glimm	Mathematical aspects of quantum field theory
Apr 14	J. Glimm	Hydrodynamics without diffusion: Theory, computation and application, PART III
Apr 14	M. Zlamal	Galerkin-finite element methods for the solution of nonlinear evolution equations, PART II
Apr 15	R. Falk	A mixed finite element method for the simply supported plate problem
Apr 16	G. Strang	Optimal design
Apr 16	F. Brezzi	Finite dimensional approximation of nonlinear problems, PART II
Apr 21	F. Brezzi	Finite dimensional approximation of nonlinear problems, PART III
Apr 21	P. Garabedian	Numerical analysis of equilibria with islands in magnetohydrodynamics
Apr 23	M. Zlamal	Galerkin-finite element methods for the solution of nonlinear evolution equations, PART III
Apr 24	F. Brezzi	Finite dimensional approximation of nonlinear problems, PART IV
Apr 28	M. Zlamal	Galerkin-finite element methods for the solution of nonlinear evolution equations, PART IV

ATTACHMENT B: Program, Abstracts of the 1-Hour Invited Lectures, Abstracts of the 30-Minute Selected Talks, and List of Registrants for the Conference on Applications of Numerical Analysis and Special Functions in Statistics

PROGRAM

p r o g r a m f o r

THE CONFERENCE ON APPLICATIONS OF NUMERICAL ANALYSIS AND SPECIAL FUNCTIONS IN STATISTICS

h e l d a t

Adult Education Center
UNIVERSITY OF MARYLAND
College Park, Maryland
October 2 - 8, 1980

s p o n s o r e d b y

U.S. Air Force Office of Scientific Research
Department of Mathematics, University of Maryland

o r g a n i z i n g c o m m i t t e e

Grace Yang	The Mathematical Statistics Program
Frank Olver	Institute for Physical Science and Technology
G.W. Stewart	Department of Computer Science

s p e c i a l y e a r i n n u m e r i c a l a n a l y s i s :

This conference is part of a Special Year in Numerical Analysis sponsored by the U.S. Air Force Office of Scientific Research and the Department of Mathematics, University of Maryland. Special Year visitors are listed below.

For further information contact Professors: P. Wolfe
Y. Yorke
I. Babuška

SPECIAL YEAR VISITORS:

Fall	Spring			
R. Askey*	G. Birkhoff	A. Harten	G. Strang	
W. Cou,*	J. Bramble	H. Keller	R. Temam	
J. Dennis*	F. Brezzi	H.-O. Kreiss	V. Thomée	
W. Gautschi*	A. Chorin	P. Lax	W. Wendlund	
J. Harrison	P. Ciarlet	M. Luskin	B. Wendroff	
S. Karlin*	J. Douglas, Jr.	A. Majda	M. Wheeler	
P. Krishnaiah*	T. Dupont	J. Nitsche	M. Zlamal	
J. Lyness*	B. Enquist	J. Oden		
C. Paige*	R. Ewing	J. Oliger		
H.O. Peitgen	R. Falk	A. Schatz		
J. Snell*	P. Garabedian	R. Scott		
L. Watson	J. Glimm	S. Smale		

*Conference speakers.

Registration

The registration booth for the Conference is located on the Main Concourse in the Adult Education Center and will be manned as follows:

Wednesday	October 1	7:30 - 9:30 P.M.
Thursday	October 2	8:30 - 12:00 A.M.
Friday	October 3	8:30 - 12:00 A.M.
Saturday	October 4	8:30 - 12:00 A.M.
Sunday	October 5	7:30 - 9:30 P.M.
Monday	October 6	8:30 - 12:00 A.M.
Tuesday	October 7	8:30 - 12:00 A.M.

Message Board

There will be a message board located at the Conference Room, #1105, for your convenience.

Additional Information

The names of the morning and afternoon chairpersons and the names and titles of the speakers giving contributed papers will be published separately, since they are not available at the time of this printing.

There will also be available a phamplet containing information on local transportation and sightseeing, and restaurants.

Thursday 10-2-80 Rm 1105

9:00 - 9:30 OPENING OF THE CONFERENCE: W.E. KIRWAN
(Chairman, Dept. of Mathematics)

WELCOMING ADDRESS: FRANK J. KERR
(Provost, Division of Mathematical & Physical Sciences & Engineering)

MORNING SESSION:

9:30 - 10:30 C.C. PAIGE, *McGill University, CANADA*
"The General Gauss-Markov Model and the Singular Value Decomposition"

10:30 - 11:00 Coffee

11:00 - 11:30 GEORGE CYBENKO, *Tufts University*
"The Efficient Solution by Orthogonalization of Linear Prediction Problems for Stationary Time Series"

11:30 - 12:00 JAMES A. CADZOW, *Virginia Polytechnic Inst.*
"Autoregressive-Moving Average Spectral Estimation: A New Effective Modeling Procedure"

AFTERNOON SESSION:

1:30 - 2:30 S. KARLIN, *Stanford University*
"A Diffusion Stochastic Model of Mathematical Genetics Involving Airy Functions"

2:30 - 3:00 Coffee

3:00 - 3:30 S.K. KATTI, *University of Missouri*
Topic: Infinite Divisibility

3:30 - 4:00 K.O. BOWMAN, *Union Carbide Corporation*
"Models for Approximating the Percentage Points of Distributions"

4:00 - 4:30 Contributed Papers

Friday 10-3-80 RM 1105

MORNING SESSION:

9:00 - 10:00 J.E. DENNIS, *Rice University*
"Inside Optimization Routines"

10:00 - 10:30 Coffee

10:30 - 11:00 FRANKLIN T. LUK, *Cornell University*
"The Communality Problem for Stieltjes Matrices"

11:00 - 11:30 ROBERT B. DAVIES, *University of California, Berkeley*
"Maximum Likelihood Estimation"

11:30 - 12:00 Contributed Papers

AFTERNOON SESSION:

1:30 - 2:30 P.R. KRISHNAIAH, *University of Pittsburgh*
"Computations of Multivariate Distributions"

2:30 - 3:00 Coffee

3:00 - 3:30 LOUIS KATES
"The Zonal Polynomials of Multivariate Analysis as Special Functions"

3:30 - 4:00 KEVIN W.J. KADELL, *University of Wisconsin*
"The Selberg Distribution"

4:00 - 4:30 Contributed Papers

Saturday 10-4-80 RM 1123

MORNING SESSION:

9:00 - 10:00 J.N. LYNESS, *Argonne National Laboratory*
"The Calculations of Trigonometric Fourier Coefficients"

10:00 - 10:30 Coffee

(SATURDAY SESSION CONTINUED ON NEXT PAGE)

10:30 - 11:00 PAUL SPECKMAN, *University of Oregon*

"Spline Smoothing and Optimal Rates of Convergence in Nonparametric Regression Models"

11:00 - 11:30 MICHAEL GHIL, *Courant Institute*

"A Stochastic-Dynamic Model for Global Atmospheric Mass Field Statistics"

Monday 10-6-80 RM 1105

MORNING SESSION:

9:00 - 10:00 J.L. SNELL, *Dartmouth University*

"Random Walks and Electric Networks"

10:00 - 10:30 Coffee

10:30 - 11:00 ROY S. FREEDMAN, *Hazeltine Corporation*

"Random Walks and Statistical Communication"

11:00 - 11:30 DAVID THOMSON, *Bell Laboratories*

"Applications of Spheriodal Wave Functions to Time Series Analysis"

11:30 - 12:00 Contributed Papers

AFTERNOON SESSION:

1:30 - 2:30 G.W. STEWART, *University of Maryland*

"Matrix Perturbation Theory and Linear Regression"

2:30 - 3:00 Coffee

3:00 - 3:30 E.J. WEGMAN, *Office of Naval Research*

"On Computer Architectures for Statistical Algorithms"

3:30 - 4:00 ASHIS SEN GUPTA, *Stanford University*

"On the Applications of Special Functions in Tests for Standardized Generalized Variances of Multivariate Normal Populations of Possibly Different Dimensions"

(MONDAY SESSION CONTINUED ON NEXT PAGE)

4:00 - 4:30 Contributed Papers

EVENING:

7:30 - 9:30 Wine and cheese garden party at the Rossborough Inn

Tuesday 10-7-80 RM 1105

MORNING SESSION:

9:00 - 10:00 W.J. CODY, *Argonne National Laboratory*

"Preliminary Report on Software for the Modified Bessel Functions of the First Kind"

10:00 - 10:30 Coffee

10:30 - 11:00 N.M. TEMME, *Mathematisch Centrum, THE NETHERLANDS*

"Incomplete Gamma Functions, Numerical and Asymptotical Aspects for Evaluation and Inversion"

11:00 - 11:30 RODERICK WONG, *University of Manitoba, CANADA*

"Some Applications of Asymptotics in Statistics"

11:30 - 12:00 Contributed Papers

AFTERNOON SESSION:

1:30 - 2:30 R.A. ASKEY, *University of Wisconsin*

"Gamma and Beta Functions From Euler to Selberg and Their Orthogonal Complements"

2:30 - 3:00 Coffee

3:00 - 3:30 CHARLES F. DUNKL, *University of Virginia*

"Discrete Orthogonal Polynomials"

3:30 - 4:00 MARIETTA J. TRETTER, *The Pennsylvania State University*

"Absolute Error Bounds for Edgeworth Asymptotic Expansions"

4:00 - 4:30 Contributed Papers

Wednesday 10-8-80 RM 1105

MORNING SESSION:

9:00 - 10:00 W. GAUTSCHI, *Purdue University*
"Special Functions: Computational Considerations"

10:00 - 10:30 Coffee

10:30 - 11:00 DONALD E. AMOS, *Sandia National Laboratories*
"Computations of the Central and Noncentral F Distributions"

11:00 - 11:30 WALTER R. NUNN, *Center for Naval Analysis*
"The Laguerre Transform"

PROGRAM SUPPLEMENT:

THE CONFERENCE ON APPLICATIONS OF NUMERICAL ANALYSIS AND SPECIAL FUNCTIONS IN STATISTICS

CHAIRPERSONS FOR THE CONFERENCE:

Thursday, Oct. 2, 1980	Morning:	Professor Diane O'Leary
	Afternoon:	Professor Peter Wolfe
Friday, Oct. 3, 1980	Morning:	Professor G.W. Stewart
	Afternoon:	Professor Paul Smith
Saturday, Oct. 4, 1980	Morning:	Professor B. Kellogg
Monday, Oct. 6, 1980	Morning:	Professor G. Yang
	Afternoon:	Professor S. Kotz
Tuesday, Oct. 7, 1980	Morning:	Professor F. Olver
	Afternoon:	Professor B. Carlson
Wednesday Oct. 8, 1980	Morning:	Professor J. Keilson

PARTIAL LIST OF CONTRIBUTED PAPERS:

Dr. E. Cuthill
David Taylor Naval Ship R & D Center

Mon. 10-6-80 4:00 — Dr. Alexander S. Elder, Aberdeen Proving Ground — "Ascending and Asymptotic Series for Squares, Products and Cross Products of the Modified Bessel Functions"

Dr. Jerry Leon Fields, University of Alberta — Topic: Convergence of an explicit sequence of rational approximations to the hypergeometric functions

$$F(b) = {}_{q+1}F_q\left\{\begin{matrix}\alpha_1,\ldots,\alpha_{q+1}\\ \beta_1,\ldots,\beta_q\end{matrix}\,\middle|\,\frac{-1}{v}\right\}$$

in the region

$$D = \{v: |\arg v| \leq \pi, \quad |\arg(1+v)| \leq \pi, \quad v \notin [-1,0]\}.$$

Mon. 10-6-80 11:45 — Dr. James W. Longley — "Modified Gram-Schmidt Process Versus Classical Gram Schmidt"

Fri. 10-3-80 11:30 — Dr. Clifford Spiegelman, National Bureau of Standards — "An Algorithm for Minimizing an Implicitly Restricted Objective Function" (with Dr. William J. Studden, Purdue Univ.)

Mon. 10-6-80 11:30 — Dr. James M. Walbert — "Use of a Continued Fraction to Evaluate the Exponential Integral in the Complex Plane"

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SPECIAL HALF-HOUR TALK:

Thurs. 10-2-80 4 - 4:30 — Dr. Richard Heilberger — "The Design and Construct of Test Data Sets for Regression Procedures" (with Dr.'s Paul F. Velleman and Agelia Ypellar)

PROGRAM SYNOPSIS

PRINCIPAL SPEAKERS

(Fifty-Minute Lectures)

R.A. ASKEY *University of Wisconsin*	"Gamma and Beta Functions From Euler to Selberg and Their Orthogonal Polynomials"
W.J. CODY *Argonne National Laboratory*	"Preliminary Report on Software for the Modified Bessel Functions of the First Kind"
J.E. DENNIS *Rice University*	"Inside Optimization Routines"
W. GAUTSCHI *Purdue University*	"Special Functions: Computational Considerations"
S. KARLIN *Stanford University*	"Various Methods for Calculating Family Correlations With Variable Family Size"
P.R. KRISHNAIAH *University of Pittsburgh*	"Computations of Multivariate Distributions"
J.N. LYNESS *Argonne National Laboratory*	"The Calculation of Trigonometric Fourier Coefficients"
C.C. PAIGE *McGill University*	"The General Gauss-Markov Model and the Singular Value Decomposition"
J.L. SNELL *Dartmouth University*	"Random Walks and Electric Networks"
G.W. STEWART *University of Maryland*	"Matrix Perturbation Theory and Linear Regression"

INVITED SPEAKERS

(Half-Hour Talks)

DONALD E. AMOS *Sandia National Laboratories*	"Computation of the Central and Noncentral F Distributions"
K.O. BOWMAN *Union Carbide Corporation*	"Models for Approximating the Percentage Points of Distributions"
JAMES A. CADZOW *Virginia Polytechnic Inst.*	"Autoregressive-Moving Average Spectral Estimation: A New Effective Modeling Procedure"

GEORGE CYBENKO *Tufts University*	"The Efficient Solution by Orthogonalization of Linear Prediction Problems for Stationary Time Series"
ROBERT B. DAVIES *Univ. of California, Berkeley*	"Maximum Likelihood Estimation"
CHARLES F. DUNKL *University of Virginia*	"Discrete Orthogonal Polynomials"
ROY S. FREEDMAN *Hazeltine Corporation*	"Random Walks and Statistical Communication"
MICHAEL GHIL *Courant Institute*	"A Stochastic-Dynamic Model for Global Atmospheric Mass Field Statistics"
ASHIS SEN GUPTA *Stanford University*	"On the Applications of Special Functions in Tests for Standardized Generalized Variances of Multivariate Normal Populations of Possibly Different Dimensions"
KEVIN W.J. KADELL *University of Wisconsin*	"The Selberg Distribution"
LOUIS KATES	"The Zonal Polynomials of Multivariate Analysis as Special Functions"
S.K. KATTI *University of Missouri*	Topic: Infinite Divisibility
FRANKLIN T. LUK *Cornell University*	"The Communality Problem for Stieltjes Matrices"
WALTER R. NUNN *Center for Naval Analysis*	"The Laguerre Transform"
PAUL SPECKMAN *University of Oregon*	"Spline Smoothing and Optimal Rates of Convergence in Nonparametric Regression Models"
N.M. TEMME *Mathematisch Centrum,* *THE NETHERLANDS*	"Incomplete Gamma Functions, Numerical and Asymptotical Aspects for Evaluation and Inversion"
DAVID THOMSON *Bell Laboratories*	"Applications of Spheroidal Wave Functions to Time Series Analysis"
MARIETTA J. TRETTER *The Pennsylvania State Univ.*	"Absolute Error Bounds for Edgeworth Asymptotic Expansions"

E.J. WEGMAN
Office of Naval Research
"On Computer Architectures for Statistical Algorithms"

RODERICK WONG
University of Manitoba
"Some Applications of Asymptotics in Statistics"

TENTATIVE SCHEDULE

This is a tentative schedule and is subject to change.

Thursday	10-2-80	9:00-9:30 A.M.	INTRODUCTION
		9:30-10:30 A.M.	C.C. Paige
		10:30-11:00 A.M.	Coffee
		11:00-12:00 A.M.	George Cybenko James A. Cadzow
		1:30-2:30 P.M.	S. Karlin
		2:30-3:00 P.M.	Coffee
		3:00-4:00 P.M.	S.K. Katti K.O. Bowman
		4:00-4:30 P.M.	Contributed Papers
Friday	10-3-80	9:00-10:00 A.M.	J.E. Dennis
		10:00-10:30 A.M.	Coffee
		10:30-11:30 A.M.	Franklin T. Luk Robert B. Davies
		11:30-12:00 A.M.	Contributed Papers
		1:30-2:30 P.M.	P.R. Krishnaiah
		2:30-3:00 P.M.	Coffee
		3:00-4:00 P.M.	Louis Kates Kevin W.J. Kadell
		4:00-4:30 P.M.	Contributed Papers
Saturday	10-4-80	9:00-10:00 A.M.	J.N. Lyness
		10:00-10:30 A.M.	Coffee
		10:30-11:30 A.M.	Paul Speckman Michael Ghil

Monday	10-6-80	9:00-10:00 A.M.	J.L. Snell
		10:00-10:30 A.M.	Coffee
		10:30-11:30 A.M.	Roy S. Freedman David Thomson
		11:30-12:00 A.M.	Contributed Papers
		1:30-2:30 P.M.	G.W. Stewart
		2:30-3:00 P.M.	Coffee
		3:00-4:00 P.M.	E.J. Wegman Ashis Sen Gupta
		4:00-4:30 P.M.	Contributed Papers
Tuesday	10-7-80	9:00-10:00 A.M.	W.J. Cody
		10:00-10:30 A.M.	Coffee
		10:30-11:30 A.M.	N.M. Temme Roderick Wong
		11:30-12:00 A.M.	Contributed Papers
		1:30-2:30 P.M.	R.A. Askey
		2:30-3:00 P.M.	Coffee
		3:00-4:00 P.M.	Charles F. Dunkl Marietta J. Tretter
		4:00-4:30 P.M.	Contributed Papers
Wednesday	10-8-80	9:00-10:00 A.M.	W. Gautschi
		10:00-10:30 A.M.	Coffee
		10:30-11:30 A.M.	Donald E. Amos Walter R. Nunn

ABSTRACTS OF THE 1-HOUR INVITED LECTURES

Abstracts of Talks by Principal Speakers for the Conference on Applications of Numerical Analysis and Special Functions in Statistics:

R.A. Askey
University of Wisconsin

"Gamma and Beta Functions From Euler to Selberg
and Their Orthogonal Polynomials"

Many of the classical orthogonal polynomials first arose in a probabilistic or statistical setting. Lagrange discovered Legendre polynomials and their recurrence relation while solving a discrete probability problem. Laplace used Hermite polynomials extensively in his book on probability theory. Fisher rediscovered the discrete Chebychev polynomials while fitting rainfall data. Fisher's representation for these polynomials was different than Chebychev's and could have led to the discovery of an important set of orthogonal polynomials related to the 6-j symbols of angular momentum theory if anyone had seriously looked at his work. From these orthogonal polynomials it is easy to find the three term recurrence relation for

$$b_n = \sum_{k=0}^{n} \binom{n}{k}^2 \binom{n+k}{k}^2$$

which was used in the first proof of the irrationality of $\zeta(3)$. In a completely different field, statistical mechanics, R.J. Baxter has recently solved another two dimensional model (the hard hexagon) and he needed the Rogers-Ramanujan identities to compute a limit associated with phase transitions. These identities were discovered by Rogers while studying some polynomials orthogonal with respect to measures that generalize the symmetric beta and normal distributions. A brief outline of these beta functions will be given and then similar integrals in several variables will be considered. After work of Wishart, Fisher, Hsu, Wilks and Ingham in statistics and Siegel in number theory, the first real break-through was made by A. Selberg in 1944, but his work was lost for almost thirty-five years. Mehta and Dyson extended Wishart's work to other classes of matrices and came up with a beautiful conjectured extension of the normal integral. This conjecture is easy to prove from Selberg's integral. Many new conjectures have been formulated in the last year or so. A few of these will be mentioned.

W.J. Cody*
Argonne National Laboratory

"Preliminary Report on Software for the Modified Bessel Functions of the First Kind"

In our experience programs which evaluate Bessel functions of various kinds are requested more frequently than programs for any other special functions. This is partially because of the importance of these functions, and partially because of the lack of high-quality transportable software for their evaluation. This report on the modified Bessel functions of the first kind is the first of a series of projected reports surveying available Bessel function software and laying the foundations for the development of a collection of transportable Bessel function programs.

After brief discussions of relevant analytic properties of the Bessel functions, important computational algorithms derived from them, and desirable properties of good numerical software, we give capsule appraisals of eleven diverse contemporary programs or program packages for the I Bessel functions. We then describe a modification of one of the more promising programs to improve its performance and extend its capabilities. Finally, this extended program and one other with similar capabilities are examined in greater detail to determine whether they are candidates for inclusion in the proposed package.

John E. Dennis
Rice University

"Inside Optimization Routines"

Applied statisticians often find library subroutines for unconstrained minimization useful. This talk will attempt to explain the ideas implemented in the best routines. We will also mention some current optimization software research directions.

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*Work performed under the auspices of the U.S. Department of Energy.

Walter Gautschi
Purdue University

"Special Functions: Computational Considerations"

We discuss computational aspects of infinite series and continued fractions in the context of evaluating special functions, in particular, probability distribution functions. Questions of major concern, with regard to infinite series, are internal cancellation of terms and stopping rules for truncating the infinite series. We introduce appropriate terminology and theory, and give applications to certain power series related to the incomplete gamma function. We then recall briefly how various types of continued fractions arise through correspondence (or association) with formal power series, or via second-order linear difference or differential equations. We advocate Euler's method of computing a continued fraction as infinite series. A new theorem is presented concerning the convergence behavior of continued fractions with real elements, and we show how it can be used to explain the empirically known effectiveness of Legendre's continued fraction for the complementary incomplete gamma function $\Gamma(a,x)$, considering that convergence, in theory, is only sublinear. We also draw attention to the computational advantages of a continued fraction of Schlomilch for the incomplete gamma function $\gamma(a,x)$.

J.N. Lyness
Argonne National Laboratory

"The Calculation of Trigonometric Fourier Coefficients"

A technique for the numerical approximation of sets of Trigonometric Fourier coefficients $\int_0^1 f(x)e^{2\pi irx}\,dx$; $r = 0,1,2,\ldots$ based on a common set function values $f(x)$, $i = 1,2,\ldots,m$ was described. The underlying theory is based on subtracting out an approximation to the truncated Euler expansion which can be integrated analytically. The method is restricted to functions having a high degree of continuity, but can be used when only irregularly spaced function values are available.

The calculation of individual Fourier Coefficients of an analytic function by using contour integration in the complex plane was also discussed briefly.

C.C. Paige
McGill University
CANADA

"The General Gauss-Markov Model and the Singular Value Decomposition"

The problem of finding the best linear unbiased estimate $\hat{\beta}$ (BLUE) for the general Gauss-Markov linear model $(y,X\beta,\sigma^2\Omega)$ may be formulated as the constrained linear least squares problem

(1) minimize u^Tu subject to $y = X\beta + Lu$,

where $LL^T = \Omega$ is the Cholesky decomposition of the given nonnegative definite symmetric matrix Ω. When Ω is positive definite L is nonsingular, and the singular value decomposition of $L^{-1}X$ could in theory be used to solve this problem. However, such an approach would not in general be numerically reliable, and is not clearly defined when L is singular as can happen in practice.

A simultaneous decomposition of L and X is suggested which is based on numerically reliable orthogonal transformations and leads immediately to the solution of (1). This decomposition is valid for all L and X with the same number of rows, and when L is nonsingular it immediately gives the singular value decomposition of $L^{-1}X$, but without using the inverse of L. Thus the decomposition is an appealing generalization of the singular value decomposition, and it solves an important class of problems as well as exhibiting their geometric structure.

J. Laurie Snell
Dartmouth College

"Random Walks and Electric Networks"

The connections between potential theory and Markov processes are well-known and has very much influenced the direction of probability theory in recent years. There are still things to learn from these connections. We show this by discussing Peter Doyle's use of Rayleigh's short-cut method to decide if discrete random walks are recurrent or transient. For this, one first identifies the walk with an electric network. Recurrence corresponds to infinite

resistance to infinity and transience to finite resistance to infinity. The network is then modified to obtain a simpler network. Two kinds of modifications are considered — shorting and cutting. Shorting decreases the effective resistance to infinity. Thus if the network, simplified by shorting, is recurrent so is the original. Cutting branches can only increase the effective resistance. Thus if the network, simplified by cutting branches, is transient so was the original this technique is illustrated by proving recurrence of simple random walk in two dimensions and transience in three dimensions.

G.W. Stewart
University of Maryland

"Matrix Perturbation Theory and Linear Regression"

This talk surveys the implications of first order perturbation theory for the linear regression problem with errors in the variables. It is shown how sets of regression diagnostics measuring the effects of these errors can be easily computed from quantities formed in the course of solving regression problems. It is also shown that under a specific model for the errors, the classical F-tests are unaffected.

ABSTRACTS OF THE 30-MINUTE SELECTED TALKS

Abstracts of Talks by Invited Speakers for The Conference on Applications of Numerical Analysis and Special Functions in Statistics:

Donald E. Amos
Sandia National Laboratories

"Computation of the Central and Noncentral F Distributions"

ABSTRACT

Recursion relations suitable for rapid, significant digit computation are derived for the cumulative distribution of $F' = (X/m)/(Y/n)$ where X is $\chi^2(\lambda,m)$, Y is $\chi^2(n)$ with X and Y independent. The cumulative for F' is given by Bayes theorem,

$$P(F' \leq f|m,n,\lambda) = \int_0^\infty G_p(X \leq fmY/n|Y = y)p(y)dy \qquad (1)$$

and where the cumulative noncentral $\chi^2(\lambda,m)$ is

$$G_p(X \leq x) = \frac{1}{2}\int_0^x \left(\frac{z}{\lambda}\right)^{p/2} e^{-(\lambda+z)/2} I_p(\sqrt{\lambda z})dz \qquad (2)$$

and the $\chi^2(n)$ density is given by

$$p(y) = \begin{cases} \dfrac{e^{-y/2}y^{-1+n/2}}{2^{n/2}\Gamma(n/2)} & y \geq 0 \\ & n \geq 1 \ . \end{cases} \qquad (3)$$

I_p is a modified Bessel function of the first kind with $p = (m-2)/2$. If we integrate (1) by parts using (2) and (3), we get

$$P(F' \leq f) = 1 - \frac{a}{2}\left(\frac{a}{\lambda}\right)^{p/2} e^{-\lambda/2}\int_0^\infty e^{-az/2} z^{p/2} I_p(\sqrt{a\lambda z})\frac{\gamma(n/2,z/2)}{\Gamma(n/2)}dz \qquad (4)$$

$$= \frac{a}{2}\left(\frac{a}{\lambda}\right)^{p/2} e^{-\lambda/2}\int_0^\infty e^{-az/2} z^{p/2} I_p(\sqrt{a\lambda z})\frac{\Gamma(n/2,z/2)}{\Gamma(n/2)}dz$$

where $\alpha = fm/n$ and the incomplete gamma functions are defined by the integrals

$$\gamma(a,x) = \int_0^x e^{-t} t^{a-1} dt \quad , \quad \Gamma(a,x) = \int_x^\infty e^{-t} t^{a-1} dt \qquad \begin{matrix} a > 0 \\ x \geq 0 \end{matrix} .$$

α, while depending upon m and n, is treated as an independent variable.

The recursive nature follows directly by applying the forward recursion relation

$$\frac{\Gamma(\beta+1,x)}{\Gamma(\beta+1)} = \frac{\Gamma(\beta,x)}{\Gamma(\beta)} + \frac{x^\beta e^{-x}}{\Gamma(\beta+1)} \tag{5}$$

to (4) since, with F the cumulative normal,

$$\Gamma(\tfrac{1}{2},x) = \sqrt{\pi}\ \mathrm{erfc}(\sqrt{x}) = 2\sqrt{\pi}\ F(-\sqrt{2x}) \tag{6}$$

and $$\Gamma(1,x) = e^{-x} \quad , \quad x \geq 0$$

can be identified. If we multiply (5) by the factors of (4) and integrate we get

$$P_{\beta+1} = P_\beta + Q_\beta \tag{7}$$

where

$$P_\beta = \frac{\alpha}{2}\left(\frac{\alpha}{\lambda}\right)^{p/2} e^{-\lambda/2} \int_0^\infty e^{-\alpha z/2} z^{p/2} I_p(\sqrt{\alpha\lambda z})\ \frac{\Gamma(\beta,z/2)}{\Gamma(\beta)}\ dz$$

$$Q_\beta = \frac{\alpha}{2^{\beta+1}}\left(\frac{\alpha}{\lambda}\right)^{p/2} \frac{e^{-\lambda/2}}{\Gamma(\beta+1)} \int_0^\infty e^{-(1+\alpha)z/2} z^{\beta+p/2} I_p(\sqrt{\alpha\lambda z})\ dz \tag{8}$$

and $P_\beta = P(F' \leq f)$ when $\beta = n/2$. In order to exploit (5), Q_β must be evaluated. A change of variables in (8), $z = t^2$, produces a form for Q_β which can be identified by

$$\int_0^\infty e^{-a^2t^2} t^{\mu-1} I_\nu(bt)dt = \frac{b^\nu}{2^{\nu+1}a^{\mu+\nu}} \frac{\Gamma\left(\frac{\mu+\nu}{2}\right)}{\Gamma(\nu+1)} \xi\left(\frac{\mu+\nu}{2},\ \nu+1,\ \frac{b^2}{4a^2}\right) , \tag{9}$$

$$a > 0,\ b > 0 \ ,$$

$\mu+\nu > 0$, where Φ is the confluent hypergeometric function defined by the series

$$\Phi(a,c,x) = \sum_{k=0}^{\infty} \frac{(a)_k}{(c)_k} \frac{x^k}{k!} \qquad c \neq 0,-1,\cdots \tag{10}$$

with

$$(a)_0 \equiv 1 \quad , \quad (a)_k = a(a+1)\cdots(a+k-1) = \frac{\Gamma(a+k)}{\Gamma(a)} \quad , \quad k \geq 1 .$$

Notice that if a is a negative integer, the series reduces to a polynomial, while for c = a, $\Phi(a,a,x) = \exp(x)$. Thus with $a = \sqrt{(1+\alpha)}/2$, $b = \sqrt{\alpha\lambda}$, $\mu = 2\beta+p+2$ and $\nu = p$ in (9), Q_β becomes

$$Q_\beta = \left(\frac{\alpha}{1+\alpha}\right)^{p+1} \frac{e^{-\lambda/[2(1+\alpha)]}}{(1+\alpha)^\beta} \frac{\Gamma(p+\beta+1)}{\Gamma(\beta+1)\Gamma(p+1)} [e^{-\rho}\Phi(p+\beta+1,p+1,\rho)] \tag{11}$$

where $\rho = \lambda\alpha/[2(1+\alpha)]$ and the scaling $e^{-\rho}$ has been introduced to eliminate the exponential growth in Φ,

$$\Phi(p+\beta+1,p+1,\rho) \sim \frac{e^\rho \rho^\beta \Gamma(p+1)}{\Gamma(p+\beta+1)} \quad \text{for} \quad \rho \to \infty \quad .$$

Now the recursion relation

$$a\Phi(a+1,c,x) = (2a-c+x)\Phi(a,c,x) + (c-a)\Phi(a-1,c,x) \tag{12}$$

with $a = p+\beta+1$, $c = p+1$, $x = \rho$ can be used to recur forward with (7) and (11), giving

$$\begin{aligned} P_{\beta+1} &= P_\beta + Q_\beta \\ A_{\beta+1} &= [A_\beta/(\alpha+1)] \cdot [(p+\beta+1)/(\beta+1)] \\ \Phi_{\beta+1} &= [(p+2\beta+1+\rho)\Phi_\beta - \beta\Phi_{\beta-1}]/(p+\beta+1) \\ Q_{\beta+1} &= \Phi_{\beta+1} A_{\beta+1} \end{aligned} \tag{13}$$

where

$$\Phi_\beta = e^{-\rho}\Phi(p+\beta+1,p+1,\rho) \quad ,$$

$$A_\beta = \left(\frac{\alpha}{1+\alpha}\right)^{p+1} \frac{e^{-\lambda/[2(\alpha+1)]}}{(1+\alpha)^\beta} \frac{\Gamma(p+\beta+1)}{\Gamma(\beta+1)\Gamma(p+1)} \quad .$$

In order to use these recurrence relations, we must have P_1, Φ_0, Φ_1, when n is even and $P_{1/2}$, $\Phi_{-1/2}$, $\Phi_{1/2}$ when n is odd. If n is even, P_1, Φ_0, Φ_1 are elementary functions. However, when n is odd, $P_{1/2}$ is a special doubly non-central t-distribution for which series representations with error bounds are given. $\Phi_{-1/2}$ and $\Phi_{1/2}$ are identified in terms of derivatives of I Bessel functions or error functions depending on whether m is even or odd. Computational techniques and stability considerations associated with the recursive computation of the Φ functions are also discussed.

A quadrature for significant digit computation of P based on (4) is also possible to cover wide ranges of parameters. The integrand is bell shaped with a single maximum at z_0. The idea is to locate z_0 from a derivative calculation, estimate the spread σ of the bell in terms of a fitted normal distribution and integrate to the left and right in steps of a 2σ or 3σ. Since the integrand is positive, no losses of significance occur due to subtraction. This procedure takes advantage of high quality routines for the special functions while computing only the dominant contributions to the sum. The computation of z_0 is facilitated by sharp algebraic estimates of transcendental functions arising from the derivative calculation.

K.O. Bowman*
Union Carbide Corporation

"Models for Approximating the Percentage Points of Distributions"

ABSTRACT

Many statistics in distributions such as the sample skewness ($\sqrt{b_1} = m_3/m_2^{3/2}$), kurtosis ($b_2 = m_4/m_2^2$), sample standard deviation, Student's t, present formidable mathematical problems especially under non-normality. Even under normality, only recently (Mulholland, Biometrika 1977) has the null distribution of $\sqrt{b_1}$ for moderate sized samples been found, although acceptable approximations have been given (see for examples, D'Agostino and Tietjen, Biometrika 1973). Mulholland had followed an early study by R. C. Gearly (Biometrika 1947), and used a careful analysis of density discontinuities by reference to an integral equation for the density for varying sample size. The application of the Gearly-Mulholland approach even to a fairly simple case such as the null distribution of kurtosis appears to be completely out of reach mathematically.

When a set of at least four moments of a statistic exists, several approximating models for the probability integral are known.

Outstanding is the Pearson system, introduced by Karl Pearson, and extensively studied from a practical point of view under the leadership of the late E. S. Pearson. The Pearson system (density y) arises from the differential equation

$$\frac{1}{y}\frac{dy}{dx} = \frac{-(x + a)}{(Ax^2 + Bx + C)}$$

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and a member is uniquely determined from (β_1,β_2), scale and location being independently adjusted. In view of the many types which can arise, tabulation of percentage points (a standard set being 0.50, 0.75, 0.90, 0.99, 0.999 and corresponding lower levels) for a limited domain of $(\sqrt{\beta_1},\beta_2)$ has only been completed in recent years, and the corresponding computer program makes heavy demand on small computer facilities.

Completely different approaches use transformation systems. The Johnson system of curves (Biometrika 1949) considers the mapping $Z = \gamma + \delta f(y)$ where $Z \varepsilon N(0,1)$. Here $f(y) = \log y$ produces the log-normal, $f(y) = \log (y/(1-y))$ produces the S_B system, and $f(y) = \sinh^{-1}(y)$ produces the S_U system. The parameters γ, δ are determined from the skewness and kurtosis of the distribution to be approximated, and involve intractable mathematics, especially in the case of S_B, which requires evaluation of four transcendental integrals.

Another transformation uses $T(x) = x^{\delta 1} - (1-x)^{\delta 2}$ (introduced by Tukey, and used on empirical data by Ramberg, Tadikamalla, Dudewicz, and Mykytka, Technometrics 1979) where x is uniform on the interval (0,1).

For the Pearson system we have introduced approximations for the standard percentage points at some 11 levels in the form of rational fractions $(\pi_1(\beta_1,\beta_2)/\pi_2(\beta_1,\beta_2))$ of the degree 3 in the parameters β_1, β_2 for a domain for which $\beta_1 < 4$. A linear formulation was used

over sets of 19 points in the (β_1,β_2) plane, the minimum error set being chosen. Rather than use a 4th degree version, we considered the segments $0 < \beta_1 < 1$, and $1 < \beta_1 < 4$, giving results with error less than 0.5%.

With the Johnson system, we have to determine as simply as possible, the values of γ and δ from those of β_1 and β_2. From a study of contours, it turns out for S_U that a rational fraction in (β_1,β_2) for the variable $\{\beta_2 - {}^1/_2(\omega^4 + 2\omega^2 + 3)\}/\beta_1$ is likely to succeed, ω being equal to $\exp({}^1/\delta^2)$. The analysis of the S_B system involving awkward quadratures is more demanding, for δ increases to ∞ as the normal point is approached, whereas γ tends to ∞ on the log-normal line. A typical model used is for example

$$\delta = (\beta_2-\beta_1-1)e^{f_1}\{\beta_{2L}-\beta_2+\log(1+\beta_1/9)\}^{f_2}$$

where f_1,f_2 are polynomial in (β_1,β_2) of degree 3. Errors in the approximation to the transformation (which once found yield all percentage points in terms of those for the normal) have been reduced to a few percentages for the three domains $0 < \beta_1 < 1$, $1 < \beta_1 < 4$, and $4 < \beta_1 < 9$.

We have confined attention to the problem of approximating percentage points for theoretical statistics whose first few moments exist; note that in many cases asymptotic series may be required using summatory techniques to establish moments evaluations. We nearly mention the corresponding problem for empirical data which though simpler from one point of view, since moments and percentiles are always available, is much more difficult when inevitable sampling errors are allowed for.

James A. Cadzow
Virginia Polytechnic Institute

"Autoregressive-Moving Average Spectral Estimation:
A New Effective Modeling Procedure"

ABSTRACT

In various signal processing applications, knowledge of the spectral density associated with a zero mean weakly stationary random time series $\{x_n\}$ plays a prominent role. This spectral density is formally given by

$$S_x(\omega) = \sum_{n=-\infty}^{\infty} r_x(n)e^{-j\omega n} \tag{1}$$

and is recognized as being the Fourier transform of the time series' covariance sequence

$$r_x(n) = E\{x_{n+k}x_k^*\} \tag{2}$$

where E denotes the expected value operator. Clearly, the determination of the spectral density entails a complete knowledge of the infinite extent covariance sequence. Unfortunately, these covariance elements are almost never known in typical applications, and, one must therefore resort to estimation techniques for determining an appropriately accurate estimate of $S_x(\omega)$. This estimate is generally based on a finite set of contiguous time series observations as designated by

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This research was supported in part by the Statistics and Probability Program of the Office of Naval Research under Contract N00014-80-C-0303

$$x_1, x_2, \ldots, x_N \tag{3}$$

Unless some constraints are imposed upon the time series' fundamental nature, however, there exists a basic incompatability in estimating spectral density (1) which depends on the infinite set of covariance elements from the finite set of observations (3). This difficulty is alleviated by postulating the following $(p,q)^{th}$ order ARMA model for the time series

$$x(n) + \sum_{k=1}^{p} a_k x(n-k) = \sum_{k=0}^{q} b_k \varepsilon(n-k) \tag{4}$$

where the unobserved excitation $\{\varepsilon(n)\}$ is taken to be a zero mean white noise series of variance one. As proven by Wald, any continuous spectral density can be approximated arbitrarily closely by an ARMA model if the order integers p and q are selected large enough.

The problem to be investigated is then that of estimating the a_k, b_k coefficients of this ARMA model from the given set of time series observations (3). Although there presently exist procedures for accomplishing this task (e.g., see refs. [1]-[5]), these procedures are not very effective in the case of small data lengths (i.e., N). In this lecture, a procedure which has been found to be effective for both short and long data lengths shall be developed. A brief outline of the procedure's salient features will now be given.

The procedure for estimating the ARMA model parameters first entails multiplying both sides of relationship (4) by $x^*(n-m)$ to yield the "basic error elements" as given by

$$e(m,n) = x(n)x^*(n-m) + \sum_{k=1}^{p} a_k x(n-k)x^*(n-m) \tag{5a}$$

$$= \sum_{k=0}^{q} b_k \varepsilon(n-k)x^*(n-m) \qquad \text{for} \quad \begin{matrix} m+1 \leq n \leq N \\ q+1 \leq m \leq N-1 \end{matrix} \qquad (5b)$$

in which * denotes the operation of complex conjugation. It is not difficult to show that if the time series is an ARMA process of order less than or equal to (p,q), then the basic error elements are each zero mean random variables over the range shown on the m and n variables. With this in mind, a logical selection for the a_k coefficients would be one that causes each of these basic error elements to be as close as possible to their expected value of zero. This objective can be achieved by minimizing the following quadratic functional

$$f(a_k) = \underline{e}^{\dagger}W\underline{e} \qquad (6)$$

where $\underline{e}$ is a (N-m)(N-q-1) × 1 column vector whose elements are appropriate arrangements of the basic error elements (5), W is a nonnegative definite square matrix, and, † denotes the operation of complex conjugate transposition. This criterion is seen to be a quadratic function of the a_k coefficients through the basic error element relationship (5a). Once the optimal set of a_k coefficients have been thus determined, the modified Welch method [6] may be applied to identify the b_k coefficients effects on the spectral estimate.

In this lecture, a more detailed development of this new ARMA model method shall be given. Moreover, the new method's performance will be empirically compared to such classical spectral estimation techniques as (i) the Box-Jenkins ARMA method, (ii) Burg's maximum entropy AR method, and, (iii) the Periodogram. In this comparison, it is found that the new method significantly outperforms the classical methods.

If this new method is to achieve its full potential, however, a number of numerically based issues need further investigation. In keeping with the spirit of this conference, these issues will be dwelled upon and suggestions solicited. Perhaps the most significant issue that needs further attention is that of selecting the weighting matrix W in criterion (6). Preliminary empirical evidence attests to the significance of this choice [7].

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George Cybenko
Tufts University

"The Efficient Solution by Orthogonalization of Linear Prediction Problems for Stationary Time Series"

Abstract

The linear prediction problem for stationary time series has traditionally been solved by forming the normal equations and solving them by either classical or fast Toeplitz algorithms. The main obstacle to using orthogonalization has been that that approach requires an order of magnitude more computations ($O(Np^2)$ as opposed to $O(Np)$).

An $O(Np^2)$ orthogonalization technique is described for general matrix orthogonalization which yields an $O(Np)$ method in the special case of linear prediction. Orthogonalization is thereby made competative with the normal equations approach.

The linear prediction problem for a stationary time series may be formulated as a matrix linear least-squares problem, where the design matrix has a Toeplitz structure.

Specifically, let $x(n)$ be a time series with only finitely many nonzero terms, so that $x(n)=0$ for $0<n<N$ say. The linear prediction problem of order p is to find coefficients $a_1,\ldots,a_p$ minimizing the expression

$$E_p = \sum (x(n) + a_1 x(n-1) + \ldots + a_p x(n-p))^2$$

where the summation is over all n.

This problem occurs in a variety of applications: Wiener filtering, stochastic model identification, speech analysis and synthesis, and geophysical signal processing, to name a few.

Letting

$$\underline{a} = \begin{bmatrix} a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_p \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} x(0) \\ \cdot \\ \cdot \\ \cdot \\ x(N) \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

$$X_p = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ x(0) & 0 & 0 & \cdots & 0 \\ x(1) & x(0) & 0 & \cdots & 0 \\ x(2) & x(1) & x(0) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & x(N-2) \\ 0 & 0 & \cdots & x(N) & x(N-1) \\ 0 & 0 & \cdots & 0 & x(N) \end{bmatrix}$$

the linear prediction problem is equivalent to solving the matrix equation

$$X_p \underline{a} = \underline{b}$$

in the least-squares sense.

The traditional approach to solving these equations has been to form the normal or Yule-Walker equations

$$X_p^T X_p \underline{a} = X_p^T \underline{b}$$

which is a simple system of equations. Because $X_p^T X_p$ has a Toeplitz structure also, it can be computed in O(Np) operations after which the pxp system requires $O(p^3)$ or $O(p^2)$ more for solution. Since N is always much larger than p, the O(Np) term dominates. Now it is well known that for small residuals, solving least-squares problems by orthogonization is more accurate than by forming and solving the normal equations. However, the orthogonalization of X_p by any of the classical methods requires $O(Np^2)$, or an order of p more, steps than the previously described approach. For this reason, orthogonalization has not been used to solve linear prediction problems.

In this paper, we present an orthogonalization technique which appears to be new, and which is $O(Np^2)$ for general unstructured matrices, and streamlines to an O(Np) method for the special Toeplitz structures that arise in linear prediction.

It is shown that this procedure is the same as the Itakura-Saito-Burg "lattice method" for linear predictive filtering and deconvolution.

Although a complete error analysis is not presently available, partial results indicate that the algorithm has good accuracy properties.

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"Maximum Likelihood Estimation"

ABSTRACT

Estimation of parameters for "non-standard" statistical problems frequently involves finding the maximum of the likelihood function or some other function related to it. Usually analytic solutions won't be possible and a numerical method will be required. While a variety of optimizing programs, available in the standard subroutine libraries can be used, the likelihood function has particular properties which it is worth taking advantage of and at the same time the statistician has particular requirements which the standard optimization programs do not necessarily provide.

This paper concentrates on the situation where the likelihood function, its first derivative and an approximation to the second derivative are available. Suppose θ denotes the vector of unknown parameters; $L(\theta)$ the log-likelihood; $\Delta(\theta) = dL(\theta)/d(\theta)$, $\Gamma(\theta) = E_\theta[\Delta(\theta)\Delta'(\theta)] = -E_\theta[d^2L(\theta)/d\theta^2]$. I will suppose $-\Gamma(\theta)$ is used as the approximation to the matrix of second derivatives.

In some problems, once L has been calculated, not too much extra work is required to obtain Δ and Γ or if L and Δ are calculated very little work is required to find Γ as well. In these situations, it seems especially worthwhile using

optimization methods that involve L, Δ, and Γ if the calculation of L is expensive, if the number of parameters is large, or if the problem is ill-conditioned. Note that $\Gamma(\theta)$ is non-negative definite (and its being singular usually indicates an error) so that one's optimize program need not take into account the possibility of $\Gamma(\theta)$ having negative or zero eigenvalues. The basic iterative step is

$$\theta_{k+1} = \theta_k + \Gamma^{-1}(\theta_k)\Delta(\theta_k)$$

but we have found that some kind of line search, based on L and Δ, is advisable.

According to maximum-likelihood theory the variance / covariance matrix of one's estimates is approximately $\Gamma^{-1}(\theta)$ so one would normally want this printed out, or rather, one wants the standard errors and correlations derived from it.

Once one has fitted a model one will frequently want to test whether it should be extended by fitting further parameters. One possibility is simply to program the extended model, fit it, look at the increase in the log-likelihood and so carry out a likelihood ratio test. But a simpler approach is to carry out Neyman's $C(\alpha)$ test. For this one still needs to calculate Δ and Γ for the full model but only for values of θ corresponding to the old model. One then calculates $\Delta' \Gamma^{-1} \Delta$ at the values of θ corresponding to the old model. If the extended model is, in fact, not necessary this term will have an approximately chi-squared distribution.

This also provides a very convenient stopping criterion for the iterative fitting process. We test the hypothesis that our current value of θ is correct by calculating $\Delta' \Gamma^{-1} \Delta$ at each step. However rather than using the critical points of the chi-squared distribution for deciding when to stop, one stops when $\Delta' \Gamma^{-1} \Delta < .001$ say. Another way of looking at this would be to say that $\Delta' \Gamma^{-1} \Delta / 2$ approximates the possible improvement in the likelihood. In any case the objective is to make sure the difference between each estimate obtained and the actual maximum-likelihood estimate is small compared with the standard error of the estimate.

If one does calculate $\Delta' \Gamma^{-1} \Delta$ at each step then the program can also be used for carrying out a $C(\alpha)$ test if it can be run for just one step.

Sometimes Δ and Γ naturally factorize: $\Delta = X'Y$, $\Gamma = X'X$ where now, perhaps, the components of Y are closely related to the original observations. In this case one can use a Householder transformation

$$X = Q \begin{pmatrix} R \\ 0 \end{pmatrix}, \quad Y = Q \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}$$

where R is upper triangular and Q orthogonal so that the basic update equation becomes $\theta_{k+1} = \theta_k + R^{-1} Z_1$; the stopping criterion becomes $\{Z_1' Z_1 < .001\}$ and $\Gamma^{-1} = (R'R)^{-1}$. This approach can be important when Γ is ill-conditioned and, in fact, there seems to be more justification for it in the present context than in the usual linear least squares situation.

This Householder approach is also particularly convenient when one wants to use recursive fitting techniques. That is when θ is broken into two (or more) parts;

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

and for each iteration of θ_2 one selects θ_1 to maximize the likelihood.

Considering the selection of the initial values from which to start the interative process. The techniques we have considered so far may not be effective if one is a long way from the correct value. Global searching may be appropriate. However we may be able to begin model fitting with a very simple model involving only one or two parameters which may be able to be estimated by other means. In particular, all "treatment effect" parameters can be set to zero. After fitting the simple model other parameters can then be added and fitted until the full model is fitted. One important attribute of the optimize program would be the ability to fix certain parameters and temporarily eliminate them from the fitting process.

Our comments have been specifically for the likelihood function. In some statistical problems it is appropriate to maximize some other function, $W(\theta)$ say. A maximum likelihood program can be applied identically to the maximization of such functions which satisfy the usual regularity conditions and also $E_\theta[dW/d\theta] = 0$, $\text{cov}_\theta[dW/d\theta, d(L-W)/d\theta] = 0$.

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"Discrete Orthogonal Polynomials"

ABSTRACT

The binomial and hypergeometric are important discrete distributions. The orthogonal polynomials for the one-variable case are the well-known Krawtchouk and Hahn polynomials. By considering some several-variable analogues one is led to another family of hypergeometric polynomials expressed as balanced ${}_4F_3$-series (appearing in physics as 6-j symbols). Special cases of these give new representations of Krawtchouk and Hahn polynomials associated to symmetric distributions (that is, binomial with $p = \frac{1}{2}$, hypergeometric $\binom{a}{x}\binom{a}{N-x}$).

1. The Hahn polynomials:

For integer parameters a,b,c we have the hypergeometric distribution $\binom{a}{x}\binom{b}{c-x}$, with $\max(0,c-b) \le x \le \min(a,c)$. The family of orthogonal polynomials for this weight is $E_m(a,b,c,x) = \sum_{j=0}^{m}(-1)^j\binom{m}{j}(b-m+1)_j(a-m+1)_{m-j}$

$\cdot(-x)_j(x-c)_{m-j} = (-1)^m(-a)_m(-c)_m\,{}_3F_2\left(\begin{matrix}-m,m-a-b-1,-x\\ -a,-c\end{matrix};1\right)$, $0 \le m \le \min(a,b,c,a+b-c)$. This is a useful notation for the Hahn polynomials (see Karlin and McGregor, Scripta Math. 26 (1961), 33-46) and the relation is

$$E_m(a,b,c,x) = (-1)^m(-a)_m(-c)_m Q_m(x;-a-1,-b-1,c)$$

(the present notation exhibits certain symmetries, and is polynomial in the parameters). These functions appear as intertwining functions on the symmetric group (consider the set of 2×2 contingency tables with row totals a and b, and

column totals c and a+b-c; for any permissible arrangement of a+b+c objects in a 2×2 array, there is a subgroup of permutations preserving rows, and another preserving columns). This fact has been used by C.D. (SIAM J. Math. Anal. 9 (1978), 627-637) to derive product and addition (b = c only) formulas.

2. Hahn polynomials in several variables:

For positive integers r, $\ell_1,\ell_2,\ldots,\ell_n$ we have the hypergeometric distribution $\binom{\ell_1}{x_1}\ldots\binom{\ell_n}{x_n}$ with $\sum_i x_i = r$. A set of orthogonal polynomials is given by

$$\prod_{j=1}^{n-1} E_{m_j}\left(\ell_j, \sum_{i=j+1}^{n} \ell_i - 2\sum_{i=j+1}^{n-1} m_i,\ r - \sum_{i=1}^{j-1} x_i - \sum_{i=j+1}^{n-1} m_i, x_j\right)$$

where $m_1,m_2,\ldots,m_{n-1}$ are nonnegative integers subject to various constraints (see §1). The product is a polynomial in $x_1,\ldots,x_{n-1}$ of degree $\sum_i m_i$ (Pearson's χ^2-statistic for the 2×n contingency table can be expressed as a weighted sum of squares of the linear polynomials). These polynomials were studied by Karlin and McGregor (pp. 261-288 in "Theory and Application of Special Functions", R. Askey, ed., Academic Press 1975) in the Q-form for a genetics application, with the ℓ_i's being replaced by negative real numbers. The present situation is associated to the permutation groups of a 2×n contingency table (column totals $\ell_1,\ldots,\ell_n$ row totals r and $\sum_i \ell_i - r$). The given basis depends on the ordering of the variables, thus a rearrangement will produce a different basis. The connection coefficients between two of these bases form a set of orthogonal transformations, one for each total degree $\sum_i m_i$. In the case n = 3, given the degree N, there is one free parameter m_1 (note $m_2 = N-m_1$), so the connection coefficients form a family of orthogonal functions in one variable. These were determined to be (C.D., Pacific J. Math to appear) balanced ${}_4F_3$-polynomials, and will be discussed in the next section.

3. The discrete ${}_4F_3$-polynomials:

For an integer N and parameters a,b,c (positive integers in the contingency table or group case), define a weight function

$$W(x;a,b,c) = \left(\frac{b+c-2x+1}{b+c-x+1}\right) \frac{(-a)_{N-x}(-c)_x(-N)_x(N-a-b-c-1)_x(-1)^x}{x!(-b)_x(x-b-c)_N}$$

(where $x = 0,1,\ldots,N$ and $a-N \leq x \leq \min(b,c,b+c-N)$ if a,b,c are integers) and the hypergeometric polynomial

$$p_k(x;a,b,c) = {}_4F_3\left(\begin{matrix}-k,k-a-c-1,-x,x-b-c-1\\ -N,-c,-a-b-c+N-1\end{matrix};1\right)$$

(a balanced series; sum of denominator parameters exceeds sum of numerator parameters by 1), a polynomial of degree k in $(x-(b+c+1)/2)^2$. The orthogonality relation is

$$\sum_x w(x;a,b,c)p_k(x;a,b,c)p_\ell(x;a,b,c) = \delta_{k\ell}/w(k;b,a,c).$$

J. Wilson (SIAM J. Math. Anal. 11 (1980), 690-701) showed that the Racah 6-j symbols could be expressed as ${}_4F_3$-polynomials (his parameters are different, and he also found continuous orthogonality relations, N replaced by a real number).

4. Special cases of the ${}_4F_3$-polynomials:

Recognizable and interesting distributions can be obtained by taking b+c+1 to be an integer, say -s, with $s \geq 0$. Neglecting constants one obtains the weight $\binom{a}{N+s+x}\binom{a}{N-x}\frac{(-c)_x}{(c+s+1)_x}$ A(s,x), $x = 0,1,\ldots,N$ where

$A(0,x) = (2-\delta_{x0})$, and for $s \geq 1$,

$A(s,x) = (x+1)_{s-1}(2x+s)/s!$, the weight being positive for $-s-1 < c < 0$.

In general one obtains polynomials not in x, but rather in $(x+s/2)^2$. However in special cases ($s = 0$, $-1 < c < 0$ or $s \geq 1$, $c = -s/2$) one can piece together two ${}_4F_3$-families to get one family of polynomials in x, a situation analogous to the quadratic transformation for Legendre polynomials, relating $P_{2n}(x)$ to $P_n^{(0,-\frac{1}{2})}(2x^2-1)$ and $P_{2n+1}(x)$ to $xP_n^{(0,\frac{1}{2})}(2x^2-1)$. The symmetric Hahn polynomials (weight $\binom{a}{x}\binom{a}{c-x}$) form an example of this. Here is the even case ($c = 2N$) (the odd case $c = 2N+1$ involves $s = 1$):

$$Q_{2n}(x;-a-1,-a-1,2N) = \frac{(\frac{1}{2})_n (N-a)_n}{(\frac{1}{2}-N)_n (-a)_n} \, {}_4F_3\left(\begin{matrix}-n,n-a-\frac{1}{2},N-x,x-N \\ -N,\frac{1}{2},N-a\end{matrix};1\right)$$

$$Q_{2n+1}(x;-a-1,-a-1,2N) = \left(\frac{N-x}{N}\right) \frac{(3/2)_n (N-a+1)_n}{(\frac{1}{2}-N)_n (-a)_n} \, {}_4F_3\left(\begin{matrix}-n,n-a+\frac{1}{2},N-x+1,x-N+1 \\ -N+1,3/2,N-a+1\end{matrix};1\right).$$

Let $a = -1$ to get the discrete Chebyshev polynomials (discrete uniform distribution), $a = 2N-\frac{1}{2}$ for the weight $\binom{4N}{2x}$, or let $a \to \infty$ to get the symmetric ($p = \frac{1}{2}$) Krawtchouk polynomials, for example

$$K_{2n}(x;\tfrac{1}{2},2N) = \frac{(\frac{1}{2})_n}{(\frac{1}{2}-N)_n} \, {}_3F_2\left(\begin{matrix}-n,N-x,x-N \\ -N,\frac{1}{2}\end{matrix};1\right).$$

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"Random Walks and Statistical Communication"

ABSTRACT

1. INTRODUCTION

We discuss an application of special function techniques to a problem that occurs in the analysis of bit error probabilities in a certain digital communication system. Let

$$\vec{A} = \sum_{i=1}^{m} e^{j\theta_i} + (w-m)$$

$$\vec{B} = \sum_{i=1}^{n} e^{j\theta_i}$$

$$A_x = \sum_{i=1}^{m} \cos\theta_i + (w-m)$$

$$B_x = \sum_{i=1}^{n} \cos\theta_i$$

$$a = |\vec{A}|$$

$$b = |\vec{B}|$$

$$P_1(m,n) = \text{Prob}(b > a)$$

$$P_2(m,n) = \text{Prob}(B_x > A_x)$$

In the above expressions, the θ_i are independent and uniformly distributed random variables on $[0, 2\pi]$; n, m, w are positive integers with n, m $\leq$ w and $j = \sqrt{-1}$. The problem is to determine P_1 (m, n) and P_2 (m, n).

These probabilities can be interpreted in the following sense. We are given two drunks: each drunk takes a unit step in an arbitrary direction. The origin is fixed at the point where drunk B starts; drunk A is given a head start of (w-n) steps in the x - direction. After a certain time, drunk B takes n random steps and drunk A takes m random steps (so the total number of steps that drunk A takes is w). P_1 (m, n) is the probability that drunk B is farther from the origin than drunk A. In this case, "farther from" is interpreted to mean that the resultant magnitude of drunk B is greater than the resultant magnitude of drunk A. If we are only interested in how far the drunks have traveled in a certain direction (without loss of generality, the x - direction), then the probability that the sum of the x - components of drunk B is greater than the sum of the x - components of a drunk A is given by P_2 (m, n). A discussion of the original interpretation in the communication system investigated is given in Section 4. It suffices at this point to denote P_1 (m, n) as the probability for the <u>non-coherent</u> case and P_2 (m, n) as the probability for the <u>coherent</u> case. Expressions for P_1 (m, n) and P_2 (m, n) will be derived below. Simplifications involve the use of Bessel function identities as well as the use of generalized functions.

2. NON-COHERENT CASE

Let $\vec{X} = \kappa e^{j\theta}$. It is well known that the characteristic function of $\vec{X}$ in polar coordinates (see reference (1)) is

$$\phi_{\vec{X}}(t) = J_0(\kappa t)$$

The addition of independent random variables yields (with k = w-m)

$$\phi_{\vec{A}}(t) = J_0^{m}(t)\, J_0(\kappa t)$$

The first factor is the characteristic function of the sum of m random phasors. The second factor is the characteristic function of the "head-start." We note that the head-start can be taken in an arbitrary direction of length k if all the other random angles are taken with respect to the first head-start angle as a reference.

A similar argument shows

$$\phi_{\vec{B}}(t) = J_0^{n}(t)$$

The density and distribution functions of a and b follow from Hankel Transformations and suitable Bessel function identities. Consequently, an expression for P_1 (m, n) is

$$\int_0^\infty dy \int_0^\infty y\, J_1(yt)\, J_0^m(t)\, J_0(\kappa t)\, dt \int_0^\infty yu\, J_0(yu)\, J_0^n(yu)\, du$$

The innermost integral (with respect to u) is the density function of b. The middle integral (with respect to t) is the cumulative distribution function of a; i.e., the probability that a is less than y.

After rearranging terms we note that

$$\int_0^\infty y^2 u\, J_1(yt)\, J_0(yu)\, dy = \dot{\delta}(u-t)$$

This formula can be proved by using the discontinuous integral of Weber and Schafheitlein. In reference (2) it is equation (29) on page 51. There seems to be a typographical error in the convergence criteria in this equation. It should read

$$Re(\nu+\mu-\rho+1)>0$$

The parameters of interest are $\rho=\mu=0$ and $\nu=1$. In this case, the hypergeometric function is completely degenerate and the Weber-Schafheitlein integral reduces to the step function represented in reference (3) as equation (9), page 406. If we differentiate this expression twice, we obtain the unit doublet as indicated above. The integral for P_1 (m, n) simplifies to

$$-\int_c^\infty J_0^{\,n}(t)\,J_0(\kappa t)\cdot\frac{d}{dt}\left[J_0^{\,m}(t)\right]\,dt$$

This use of the Weber-Schafheitlein integral seems to have first been used by Doyle (see reference (4)). Kluyver uses the integral to derive an expression for the density function of b (see reference (3), page 419).

After an integration by parts we obtain

$$P_1(m,n)=\frac{n}{n+m}\left[1-\kappa\int_0^\infty J_1(\kappa t)\,J_0^{\,m+n}(t)\,dt\right]$$

The last integral cannot be simplified further. For n and m large we can obtain asymptotic expressions for the density of a and b: b is asymptotically Rayleigh distributed and a is asymptotically Rice-Nakagami distributed. We find that

$$P_1(m,n) =_{asym.} \frac{n}{n+m} e^{-\kappa^2/(n+m)}$$

a result which agrees exactly for k=0.

3. COHERENT CASE

In this case, we do not deal with magnitudes of phasors and do not calculate densities with Hankel Transforms. Consequently, the following characteristic functions for A_x and B_x are

$$\phi_{A_x}(t) = J_0^m(t)\, e^{i\kappa t}$$

$$\phi_{B_x}(t) = J_0^n(t)$$

After taking Fourier Transforms and making use of the identity

$$\frac{1}{\pi}\int_0^{\infty} \cos xt\, dx = \delta(t)$$

we find that

$$P_2(m,n) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_0^{\infty} \cos t(u-\kappa)\, J_0^m(t)\, dt\, du$$

$$- \frac{1}{\pi^2}\int_{-\infty}^{+\infty}\int_0^{\infty}\int_0^{\infty} \frac{\cos t(u-\kappa)\, J_0^m(t)\, J_0^n(y)\sin uy\, dy\, dt\, du}{y}$$

which after simplification yields

$$P_2(m,n) = \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty} \frac{J_0^{m+n}(x)\sin(\kappa x)}{x}\,dx$$

For n and m large, a and b are respectively asymptotically non-zero and zero mean gaussian distributed. Consequently

$$P_2(m,n) =_{asym.} \frac{1}{\sqrt{2\pi\sigma}}\int_0^{\infty} e^{-(x-\kappa)^2/2\sigma^2}\,dx$$

with $\sigma^2 = (m+n)/2$.

This result also agrees with the exact expression for k=0.

4. INTERPRETATION FOR STATISTICAL COMMUNICATION

In our system either a "mark" or "space" signal is transmitted, both events being equally likely. We assume a mark is transmitted. The probability of error is given by Prob (receive space given that a mark was transmitted). This probability depends not only on the transmission media but also on the encoding and modulation schemes. We will assume that a mark (or space) is encoded onto a set of w waveforms, with each waveform suitably represented as a phasor. We will assume that a phase reference is known at the receiver. The effect of noise is to change the amplitude and phase of a particular waveform. Intuitively, the more waveforms, the less likely it will be that noise will disrupt all w waveforms. At the receiver, each particular waveform will be detected and "hard-limited" in that the magnitude of a received phasor will be normalized to unity if that

magnitude exceeds a certain threshold. This minimizes the effects of strong interference; information is still contained in the phase. We assume that in the w mark waveforms, m contain interference and in the w space waveforms, n contain interference. In the non-coherent case, a mark is decided if the magnitude of the resultant phasor sum for the mark waveforms is greater than the resultant phasor sum for the space waveforms. In the coherent case, we compare the sum of the values of the components in each waveform. The error probabilities are then given by P_1 (m, n) and P_2 (m, n).

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"A Stochastic-Dynamic Model for Global Atmospheric Mass Field Statistics"

ABSTRACT

A model that yields the spatial correlation of atmospheric temperature data has been developed. It involves the solution of the potential vorticity equation forced by random noise:

$$(\nabla^2 - C_0 \sin^2\theta)\,\phi(\lambda,\theta;\omega) = F(\lambda,\theta;\omega) \qquad (1)$$

where ∇^2 is the laplacian operator in the unit sphere, λ and θ are longitude and latitude, ϕ is the temperature and F is white noise corresponding to a random realization ω.

The spatial correlation Γ is then computed from

$$\Gamma(\lambda_1,\theta_1;\lambda_2,\theta_2) = E\left[\phi(\lambda_1,\theta_1;\omega)\,\phi(\lambda_2,\theta_2;\omega)\right] \qquad (2)$$

where E is the expected value.

Three methods of solution have been tested. In the first method, Eq. (1) was solved by expansion in spherical harmonics and the correlation function was computed analytically using the expansion coefficients. In the second method, the finite difference equivalent of Eq. (1) was solved using a Fast Poisson

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Solver. The correlation function Γ was computed using stratified sampling of the individual realizations of $F(\omega)$ and hence of $\phi(\omega)$. In the third method an equation for Γ was derived from Eq. (1) and solved directly in finite differences by succesive applications of the Fast Poisson Solver. The three methods were compared for accuracy and efficiency, and the third method was chosen as clearly superior.

The results agree well with the latitude dependance of observed atmospheric correlation data. The value of the parameter C_0, chosen by best fit to the data, is close to the value expected from dynamical considerations.

These results provide the basis for an optimal choice of coefficients for statistical analysis of atmospheric data.

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"On the Applications of Special Functions in Tests for Standardized Generalized Variances of Multivariate Normal Populations of Possibly Different Dimensions"

ABSTRACT

1. Summary. The concept of Standardized Generalized Variances (SGVs) is introduced. Several new problems of multivariate statistical inference are formulated on the basis of these SGVs. It is shown that in addition to providing several new statistical tests, many existing problems of multivariate tests of significance can be regarded as special cases of these formulations and can also be extended to their full generalities. The null and non-null distributions of these test criteria are thus of vital importance. Considering multivariate normal populations with general and equi-correlated covariance matrices, these distributions are deduced in computable forms in terms of a variety of Special Functions, e.g., Pincherle's H-function, Meijer's G-function, Kummer's function, Whittaker's function, Riemann's Zeta function and Psi-function. The highly desirable property of unbiasedness is also established for most of the above test criteria. Finally, applications of the above tests to a wide spectrum of applied research are also illustrated by examples taken from the existing literature.

2. Introduction. Let X be a p-dimensional random vector variable with dispersion matrix Σ. Two well-known measures of multidimensional scatter, obtained by generalizing the variance, the univariate measure, are Σ and the generalized variance (GV), $|\Sigma| = \det(\Sigma)$, introduced by Wilks (1932). For multivariate normal populations, Likelihood Ratio Tests (LRTs) for Σs, of course of same dimensionalities, and some optimum properties of these tests are known. But, when multi-dimensional scatter of populations of different dimensions need to be compared, these tests cannot be defined. However, using $|\Sigma|^{1/p}$, which we will nomenclature as Standardized Generalized Variance (SGV), such comparisons become meaningful. Since $|\Sigma|$ represents the volume in p-dimensions, note that $|\Sigma|^{1/p}$ becomes a measure so scaled as to become comparable with scatter for a scaler random variable. Apart from this generality, need for tests of generalized variances have been also felt, on its own right. $|\Sigma|$, being a scalar, is more suitable and easier to work with than the matrix Σ. Hoel (1937) was probably first to realize this need and later Eaton (1967) studied some problems of statistical inference associated with a single GV. The GV has been extensively used in applied research, e.g., by Goodman (1966) in Agricultural Statistics, Gnanadesikan and Gupta (1970) in Ranking and Selection, Arvanitis and Afonja (1971) in Sample Survey, etc. While the estimation, e.g., van der Vaart (1965), Shorrock and Zidek (1976), and the distribution, e.g., Bagai (1965), Mathai (1972) have been studied in some detail, little seems to be known about tests for GVs. This paper attempts to bridge that gap via the extensive use of Special Functions.

Suppose $\underset{\sim}{X}_i$s are independently distributed as $N_{p_i}(\mu_i,\Sigma_i)$, Σ_i being general dispersion matrices, i = 1,...,k. LRTs are deriv i for $H_{01} : |\Sigma_i|^{1/p_i} = \sigma_0^2$ (given) > 0, for some fixed i; $H_{02} : |\Sigma_i|^{1/p_i} = |\Sigma_j|^{1/p_j}$, for some fixed i and j and finally $H_{03} : |\Sigma_i|^{1/p_i}$ all equal, i = 1,...,k against appropriate two-sided alternatives. The test criteria turn out to be quite elegant multivariate analogues to those in the univariate cases. The special case, when Σ_i s are equi-correlated matrices, i.e., $\Sigma_i = ({}_i\rho_{uv}) = \Sigma_{\rho_i}$, where ${}_i\rho_{uv} = 1$, u = v, and ${}_i\rho_{uv} = \rho_i$, $u \neq v$, is next considered. As Anderson (1963) has pointed out, the statistical inference dealing with correlation matrices can become much more complicated compared to those dealing with covariance matrices. However, such equi-correlated structure as considered above is of extreme importance [see, e.g., Kshirsagar (1978), p. 227] and has extensive applications [see, e.g., Mitra and Ling (1979)] in applied research. After exhibiting the shortcomings of the LRT in this case, some new tests, including one based on the smallest chracteristic root of the dispersion matrix, are given in this paper.

The solutions to the distributional problems associated with the various test statistics considered above need extensive use of Special Functions. The exact distributions for both the null and non-null cases are presented for most of the above test criteria. The percentage points of these distributions can be obtained from existing mathematical tables since the distributions are represented in suitable computable forms. Examples of construction of tables and

the general procedure of obtaining them for such computable forms of the distributions exist in current literature, e.g., Mathai (1979). Further, many of the existing tables can also be exploited to give the percentage points. Large sample approximations to the above distributions are also presented.

Considering general Σ_i of equal dimensions, for H_{01} and H_{02}, it is shown that the 'modified' LRTs are unbiased--a result parallel to Sugiura and Nagao (1968) on tests of covariance matrices. Total and partial unbiasedness of some of the tests proposed for the equi-correlated case are also established. It is shown that the same invariant measure under the full linear group that was exploited to prove the unbiasedness in the cases of tests for covariance matrices can also be exploited here to establish the unbiasedness of tests for SGVs.

3. Applications. In addition to the mathematically interesting nature of the problem and the applications cited above, there lies a rich fertile area for numerous applications of the SGVs. In fact, wherever variance is employed for univariate situations, SGVs seem to be applicable for the multivariate situations. Some examples are cited below.

(a) Multivariate Quality Control. It is well known [e.g., see Steyn (1978)] that testing H_0 : the population mean vector $\underset{\sim}{\mu}_r$ of $\underset{\sim}{X}$, $\underset{\sim}{X}_r \sim N_p(\underset{\sim}{\mu}_r, \Sigma)$, remains constant during the sampling process against the alternative that $\underset{\sim}{\mu}_r$ varies during the process, is equivalent to

testing $H_0 : GV = |\Sigma|$ against $H_1 : GV = |\Sigma^*|$, where $\Sigma^* = (I + 2D/n)\Sigma$, $D = \Sigma_{r=1}^{n} \underset{\sim}{\mu}_r \underset{\sim}{\mu}_r' \Sigma^{-1}$, n being the sample size. One of the many applications of this result and hence test for SGV can be seen in multivariate quality control.

(b) Generalized Canonical Variable (GCV) Analysis. When the original vector can be divided into $k \geq 2$ mutually exclusive groups, Anderson (1958) proposed GCVs to be obtained by minimizing their GV. Steel (1951) and Kettering (1969) (in his Ph.D. thesis) have attempted to construct such GCVs. However, no results on statistical inference associated with these GCVs are available. Gnanadesikan (1977), drawing from a well-known example in psychometry, posed the problem of selection among GCVs obtained by different types of grouping. The problem with its extension in full generality boils down to tests of SGVs and hence can be tackled by the methods outlined in this paper.

(c) Generalized Homogeneity of Multi-dimensional Scatter. Dyer and Keating (1980) were interested in the homogeneity of variances of the sealed bids of 'five' Texas offshore oil and gas leases. A glance at their data reveals that the 'five' leases are actually five groups with different number of components. Treating the data as in an univariate set-up, they proceeded to test for the homogeneity of variances. However, it seems more appropriate to consider the groups as vector variables and test for homogeneity of 'variances' of these groups, which we can term as 'Generalized Homogeneity.' This will be equivalent to testing homogeneity of SGVs, which is precisely H_{03} defined above.

4. Distributions and Associated Special Functions. First consider general Σ_i s. Let S_i s be sample sums and product matrices, $N_i^{2p_i} S_{i0} = |S_i|$, N_i being the sample size, $i = 1,\ldots,k$. Consider the setup of Section 2. Let ω_j be the critical region of the LRTs or equivalent criteria for testing H_{0j} against both-sided alternatives, $j = 1,2,3$. Two-stage maximum likelihood estimation and some judicious transformations are employed to provide the following results.

$$\omega_1 : \lambda_1 = |S_i|^{1/p_i} > a_0 \text{ or } < a_1;\ a_0, a_1 \text{ being constants.}$$

$$\omega_2 : \lambda_2 = |S_i|^{1/p_i} / |S_j|^{1/p_j} > b_0 \text{ or } < b_1;\ b_0, b_1 \text{ being constants.}$$

$$\omega_3 : \lambda_3 = \prod_{i=1}^{k} \{S_{i0}^2 / (\Sigma N_i p_i\ S_{i0}^2 / \Sigma N_i p_i)\}^{N_i p_i / 2} < \eta_0 \text{ (constant).}$$

Through the use of Calculus of Residues, the exact null and non-null distributions of λ_1 is given in computable forms of G-functions, those of λ_2 in computable forms of H-function and those of $u = \zeta\lambda_3$, ζ being a constant, for all N_i and k but $p_i = 1$ or 2 as,

$$f_{\nu,\underset{\sim}{a}}(u) = [\Gamma(\tfrac{\nu}{2}) / \Pi\Gamma(\tfrac{\nu}{2} a_j)] C_{\underset{\sim}{a}}^{\nu/2} (\Pi a_j^{-\frac{1}{2}}) [(2\pi)^{(n-1)/2} / \Gamma(\tfrac{n-1}{2})] u^{k-1}$$

$$\cdot (-\log u)^{(n-3)/2}\ \xi_{\underset{\sim}{a}}(u) \quad , \quad 0 < u < 1$$

where $\xi_{\underset{\sim}{a}}(u) = 1 + \Sigma_{r=1}^{\infty} v_r(\underset{\sim}{a})(-\log u)^r$ if $e^{-2\pi\beta} < u < 1$ and $v_r(\underset{\sim}{a})$, a function of Bernoulli numbers, ν, $C_{\underset{\sim}{a}}$, and β are obtainable when N_i, k and p_i s are specified. Tables for the distribution of u are given in Dyer and Keating (1980). Usual large-sample χ^2 approximations hold

for $-2 \log_e \lambda_j$, $j = 1,2,3$. Additionally, normal and Gamma approximations for $s_{i0}^2 / |\Sigma_i|^{1/p_i}$ are reviewed. An F-approximation is provided for the distribution of λ_2. For λ_3, an approximation is given in terms of Bartlett's distribution. These approximations turn out to be exact when p_i s are any combinations of 1 s or 2 s.

Next the equi-correlated case is considered. The LRT for $H_0 : \rho_i = \rho_{i0}$ against $H_1 : \rho_i \neq \rho_{i0}$ (i fixed) is derived. The MLE for ρ_i is shown to have some undesirable properties. A test based on truncated Best Unbiased Estimator for ρ_i is provided. The distributions of the test statistic for both the null and non-null cases are represented in terms of Kummer's function and Whittaker's function. The LRT for H_{01} is given. Additionally, a simpler test is provided through a characterization of the problem in terms of the smallest eigenvalue of the covariance matrix. LRTs and modified tests using Isotonic regression techniques are also provided for H_{02} and H_{03}.

Total and partial unbiasedness of most of the tests discussed above are also established.

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"The Selberg Distribution"

ABSTRACT

In 1944, A. Selberg [4] evaluated an important multivariate extension of the beta integral. He showed that

$$(1)\qquad \int_0^1 \cdots \int_0^1 \prod_{i=1}^{n} t_i^{x-1}(1-t_i)^{y-1} \left| \prod_{1\le i<j\le n} (t_i-t_j) \right|^{2k} dt_1 \cdots dt_n$$

$$= \prod_{j=1}^{n} \frac{\Gamma(x+(j-1)k)\,\Gamma(y+(j-1)k)\,\Gamma(1+jk)}{\Gamma(x+y+(n+j-2)k)\quad \Gamma(1+k)} \quad ,$$

where $Re(x) > 0$, $Re(y) > 0$ and $Re(k) > -\frac{1}{n}, -\frac{Re(x)}{n-1}, -\frac{Re(y)}{n-1}$. Upon normalization, (1) gives the joint density function of our principle object of study: the Selberg distribution with parameters x, y, k and n. Some important limiting distributions are as follows. Let $x = y$ and $y \to \infty$. Then (1) becomes

$$(2)\qquad \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} e^{-\frac{1}{2}\sum_{i=1}^{n} t_i^2} \left| \prod_{1\le i<j\le n} (t_i-t_j) \right|^{2k} dt_1 \cdots dt_n$$

$$= \prod_{j=1}^{n} \frac{\Gamma(1+jk)}{\Gamma(1+k)} \quad ,$$

$Re(k) > -\frac{1}{n}$, which was studied by Mehta and Dyson [3]. For $k = \frac{1}{2}$, $k = 1$ and $k = 2$, this corresponds to the distribution of the eigenvalues of orthogonal, hermitian and symplectic matrices, respectively.

Letting $y \to \infty$ in (1) yields

$$(3) \qquad \int_0^\infty \cdots \int_0^\infty \prod_{i=1}^{n} t_i^{x-1} e^{-t_i} \left| \prod_{1 \le i < j \le n} (t_i - t_j) \right|^{2k} dt_1 \cdots dt_n$$

$$= \prod_{j=1}^{n} \frac{\Gamma(x + (j-1)k)\, \Gamma(1 + jk)}{\Gamma(1 + k)} ,$$

where $\mathrm{Re}(x) > 0$ and $\mathrm{Re}(k) > -\frac{1}{n}$, $-\frac{\mathrm{Re}(x)}{n-1}$. For $k = \frac{1}{2}$, this corresponds to the distribution (see Anderson [1; ch. 13]) of the eigenvalues of a random matrix from the Wishart distribution with mean vector 0 and spherical dispersion matrix.

For $n = 1$, (1), (2) and (3) reduce to the univariate beta, normal and gamma or chi-square distributions, respectively. The most fundamental of these is the beta distribution, since it has 2 parameters and includes the others as limiting cases. We are led to study the Selberg distribution where x, y, $2k$ and n are positive integers. For $n = 1$, this is the distribution of the x^{th} order statistic $u_{(x)}$ from a sample of $x + y - 1$ iid $u(0, 1)$ random variables. We shall generalize this to Selberg's distribution. Let

$$(4) \qquad M = n(x + y - 1) + 2k\binom{n}{2} .$$

Decompose the set of integers from 1 to M as a disjoint union

$$\{1, \cdots, M\} = \{m_1, \cdots, m_n\} \cup \bigcup_{i=1}^{n} (S_i \cup L_i) \cup \bigcup_{1 \le i < j \le n} B_{ij} ,$$

where

$$|S_i| = x - 1, \quad |L_i| = y - 1, \quad 1 \le i \le n \quad \text{and} \quad |B_{ij}| = 2k, \quad 1 \le i < j \le n .$$

The number of ways in which this can be done is given by the multinomial coefficient

$$\binom{M}{1,\cdots,1,x-1,\cdots,x-1,y-1,\cdots,y-1,2k,\cdots,2k}$$

$$= \frac{M!}{[(x-1)!\ (y-1)!\,]^n\ \ (2k!)^{\binom{n}{2}}} \quad ,$$

which we denote by $\binom{M}{*}$. Let $T(x,y,k,n)$ denote the number of these choices which also satisfy (inequalities with sets hold for all elements)

(5) $$S_i < m_i < L_i\,, \quad 1 \le i \le n \quad \text{and} \quad m_i < B_{ij} < m_j\,, \quad 1 \le i < j \le n \ .$$

The condition (5) implies $m_i < m_j$ for $1 \le i < j \le n$. Let $I(x,y,k,n)$ denote Selberg's integral (1) . Then

(6) $$\frac{I(x,y,k,n)}{n!} =$$

$$\int_{0<t_1<t_2<\cdots<t_n<1} \cdots \int \prod_{i=1}^{n} t_i^{x-1}(1-t_i)^{y-1} \prod_{1\le i<j\le n}(t_j-t_i)\,dt_1\cdots dt_n$$

corresponds to the distribution of the order statistics from Selberg's distribution. We have

(7) $$\frac{I(x,y,k,n)}{n!} = \frac{T(x,y,k,n)}{\binom{M}{*}} \quad ,$$

so that the problem of evaluating (1) is equivalent to the combinatorial problem of evaluating $T(x, y, k, n)$. Let

$$u_{(1)} < u_{(2)} < \cdots < u_{(M-1)} < u_{(M)}$$

be the order statistics of a sample of M iid $u(0, 1)$ random variables. Choose one of the objects counted by $T(x, y, k, n)$ at random, with each equally likely to be selected. Then

(8) $$\vec{t} = (t_1, \cdots, t_n)$$
$$= (u_{(m_1)}, u_{(m_2)}, \cdots, u_{(m_n)})$$

is distributed as the order statistics of Selberg's distribution (corresponding to (6)). Let $\sigma \in S_n$ be chosen at random, with each permutation equally likely to occur. Then

(9) $$\sigma(\vec{t}) = (u_{(m_{\sigma(1)})}, \cdots, u_{(m_{\sigma(n)})})$$

has Selberg's distribution. This fact and (7) both follow by generating the random variables in (6) from an acceptance-rejection procedure

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Louis Kates

"The Zonal Polynomials of Multivariate Analysis as Special Functions"

ABSTRACT

1. Introduction

The special functions which have received the most attention in multivariate analysis are the zonal polynomials. Modulo the orthogonal group, they are roughly the eigenfunctions of matrix multiplication. They provide the natural group theoretic basis for the space of complex-valued analytic functionals of an n x n matrix which are both left and right invariant under the orthogonal group. Since this space is the home of the multivariate Gaussian density as well as others, its natural decomposition is necessarily of interest.

The ease of dealing with zonal polynomials would be enhanced considerably if the abstract eigenfunction property were augmented with a concrete and explicit workable formula. The desirability and difficulty of obtaining such an expression

has been discussed on several occasions (James, 1975), (MacLaren, 1975). It is our purpose to give such an expression and note how it arises from considering zonal polynomials as special functions defined in the group theoretic sense of zonal spherical functions on symmetric space. The discussion and results presented here have previously appeared in the author's Ph.D. thesis (Kates, 1980). An attempt has been made to keep the exposition at a fairly elementary level.

2. Fourier Analysis

We take the viewpoint of Fourier analysis on groups. This approach has not previously been taken in the statistical literature. It is nevertheless a quite natural and direct approach. We proceed partly by example.

Let $G = \{g_1, g_2, g_3, g_4\}$ be a four element commutative group. Let $C(G) = \{f : G \to C\}$ denote the set of complex-valued functions on G. Each function f is defined by a four-tuple $(f(g_1), f(g_2), f(g_3), f(g_4))$ showing that C(G) is a four dimensional vector space. The operators $T_{g'}$ which act on C(G) are linear and are defined as

$$T_{g'} f(g) = f(gg').$$

They have the effect of translating the graph of f by g'. The commutativity of G implies the commutativity of the T_g.

The T_g are individually diagonalizable commuting 4 by 4 matrices and so can be simultaneously diagonalized. Their simultaneous eigenvectors are called zonal spherical functions and form the natural basis for Fourier analysis on C(G).

If G is non-commutative the T_g no longer commute so we cannot simultaneously diagonalize them. However, if we are only interested in a subspace V of C(G) then it may be true that the T_g operators, suitably restricted to V, are commutative.

Now suppose that G is not a four element commutative group but rather the n by n nonsingular real matrices regarded as a (noncommutative) group under matrix multiplication and inversion. Within C(G), the complex-valued functions on G, let V be that subspace consisting of those functions which are:

(i) expressible as a power series in the n^2 elements of their argument

(ii) orthogonally bi-invariant, namely f(XXK) = f(X), for all orthogonal H and K.

Let P be the projection of the vector space C(G) onto the subspace V. Then the operators PT_g acting on V commute and

their simultaneous eigenfunctions are called zonal spherical functions of the pair of groups: real non-singulars/orthogonals. They are also called zonal polynomials.

3. An Explicit Formula

Let $\kappa = (k_1, k_2, \ldots, k_u)$ where $k_1 \geq k_2 \geq \ldots \geq k_n \geq 0$ are integers. Suppose $\Delta_i = \Delta_i(N^T X^T X N)$ is the determinant of the upper left i x i submatrix of $N^T X^T X N$ where X is a nonsingular n x n matrix and N is an n x n random matrix whose entries are independent Gaussian random variables, each with mean 0 and variance 1. The orthogonal bi-invariance property of zonal polynomials implies that they depend on X only through $X^T X$ so we regard them as functions of $X^T X$ in what follows. Our aim is to show that

$$(*) \qquad Z_\kappa(X^T X) = E(\Delta_1^{k_1-k_2} \Delta_2^{k_2-k,} \ldots \Delta_{n-1}^{k_n-k_{n-1}} \Delta_n^{k_n})$$

by showing that (*) satisfies the eigenfunction property and forms a basis for V. Here E means expectation.

To evaluate any given zonal polynomial multiply out the integrand giving a polynomial in independent Gaussian random variables, each with mean 0 and variance 1. Since the $2m^{th}$ moment of such a random variable is $(2m)!/(2^m m!)$ and all odd mo ts are 0, substitution yields the polynomial

explicitly. Alternatively, one could Monte Carlo the function

$$\Delta_1^{k_1-k_2} \Delta_2^{k_2-k_3} \cdots \Delta_n^{k_n} .$$

Since zonal polynomials are eigenfunctions, their normalization is unimportant. Thus with change of normalization (*) holds true when the probability distribution of N is replaced by the uniform probability distribution over the orthogonal group (also called Haar probability measure). This renormalization is such that the zonal polynomial equals 1 when evaluated at the identity. We denote it $C_K^*(X^TX)$. In one of these two forms the formula can be used to prove many additional facts about zonal polynomials as well as to reprove known results in a more direct fashion.

FOOTNOTES

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TOPIC: Infinite Divisibility

ABSTRACT

This topic covers a set of three papers on the topic of infinite divisibility. Infinite divisibility has always been tucked away in the jargon of complex variables. If we stick to integer valued variables, then this tipic can be handled through very simple algebra and through the use of computers whenever necesary. A necessary and sufficient condition is that a series of determinants be nonnegative. Thus the user can test for infinite divisibility be computing a bunch of these determinants and see if they are nonnegative. In the papers, we give proofs of infinite divisibility using these determinants. We also give sufficient conditions. One sufficient condition is, "If P_i, $i = 0,1,\ldots$ are such that P_i/P_{i-1} are monotone nondecreasing, then the distribution described by P_i is infinitely divisible." A side issue resulting from this extreme simplicity and "computerizing of the problem and pulling it out of the range of the abstract complex variables" is that we are now trying to test for independence of observations in successive plots in a farm using the first few determinants as our test statistic.

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"The Communality Problem for Stieltjes Matrices"

ABSTRACT

The Communality Problem is very important in Factor Analysis. The problem is that of reducing the diagonal elements of a given correlation matrix so that the resulting matrix will be positive semidefinite and of minimum rank. The new diagonal elements are called the communalities. We defind a correlation matrix as a symmetric and positive semidefinite matrix with unities along the diagonal and fractional numbers between -1 and +1 in the off-diagonal positions.

Many researchers have studied the Communality Problem (for a detailed set of references, see Harman [1]). However, no effective solution procedures have yet been devised. In this paper, we propose a variant problem and give an algorithm for its solution. We prove that a solution to our problem also solves the Communality problem if the given matrix is Stieltjes.

We can state the Communality Problem as follows:

Problem 1

Given a correlation matrix R, find a non-negative diagonal matrix D such that

(i) R - D is positive semidefinite, and

(ii) rank (R - D) = min. □

Let us examine the variant problem:

Problem 2

Given a correlation matrix R, find a non-negative diagonal matrix D such that

(i) R - D is positive semidefinite,

(ii) rank (R - D) = min., and

(iii) rank (R - D) + rank (D) = rank (R). □

Condition (iii) is the additional constraint that we have imposed.

For convenience, we use the following notations:

Notations

We let CP(A) denote Problem 1 with A as the given correlation matrix, and let P(B) denote Problem 2 with B as the given correlation matrix. □

For a given correlation matrix R, let D_1 and D_2 be the nonnegative diagonal matrices that solve CP(R) and P(R), respectively. The extra condition for P(R) implies that

$$\text{rank } (R - D_1) \leq \text{rank } (R - D_2).$$

We can easily construct an example for which the inequality above is strict.

Let us describe an algorithm for solving P(R), where R is a given correlation matrix. Let an eigenvalue decomposition of R be

$$R = Q\Sigma Q^t,$$

where Q is orthogonal and

$$\Sigma = \begin{pmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{pmatrix}$$

with Σ_1 a nonsingular diagonal matrix of order r.

Let

$$Q = (Q_1, Q_2),$$

where Q_1 is n x r.

Define the index set

$$Z = \begin{cases} \{i \mid \text{row } i \text{ of } Q_2 \text{ consists of all zeros}\}, & \text{if } r < n. \\ \{1,2,\ldots,n\} & \text{if } r = n. \end{cases}$$

If Z is empty, we cannot reduce the rank of R without giving the resulting matrix a negative eigenvalue. Hence we assume that Z is nonempty. Let

$$Z = \{i(1), i(2), \ldots, i(\ell)\}.$$

For $j = 1,2, \ldots, \ell$, we can find a vector $\underset{\sim}{x}_{i(j)}$ such that

$$R\underset{\sim}{x}_{i(j)} = \underset{\sim}{e}_{i(j)}.$$

The general solution is

$$\underset{\sim}{x}_{i(j)} = Q_1\Sigma_1^{-1} Q_1^t \underset{\sim}{e}_{i(j)} + \underset{\sim}{w},$$

where $\underset{\sim}{w} \in N(R)$.

Let Y be an d x d matrix with its (j,k) element equal to the i(j)-th element of $\underset{\sim}{x}_{i(k)}$. Note that the elements of Y are not affected by the choice of $\underset{\sim}{w}$ in the previous equation, as $w_{i(j)} = 0$ for $j = 1,\ldots,\ell$.

Let I be the set of indices such that for $i,j \in I$

$$y_{ij} = 0 \qquad \text{for } i \neq j.$$

Construct the diagonal matrix

$$D = \sum_{j \in I} \alpha_{i(j)} e_{i(j)} e_{i(j)}^t,$$

where

$$\alpha_{i(j)} = \frac{1}{y_{jj}}.$$

Note that $\alpha_{i(j)}$ is positive because

$$y_{jj} = e_{i(j)}^t Q_1 \Sigma_1^{-1} Q_1^t e_{i(j)}$$

and

$$e_{i(j)} \notin N(R).$$

It is easy to check that

$$(R - D)x_{i(j)} = 0 \qquad \text{for } j \in I.$$

We can also show that the matrix R - D is positive semidefinite and that

$$\text{rank } (R - D) = \text{rank } (R) - \text{rank } (D).$$

As our goal is to minimize rank (R - D), we want to determine the index set I of maximum size. It is known, however, that determining such an I requires work which is exponential in ℓ, the size of Z. Fortunately, ℓ is usually very small for the matrices of the Communality Problem.

We next consider the case when our correlation matrix is a Stieltjes matrix, i.e. a positive definite matrix with nonpositive off-diagonal elements. Let D be a nonnegative diagonal matrix which solves P(R). We are going to show that D is also a solution to CP(R).

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"The Laguerre Transform"

ABSTRACT

A novel transform is presented which maps continuum functions (such as probability distributions) into discrete sequences and permits rapid numerical calculation of convolutions, multiple convolutions, and Neumann expansions for Volterra integral equations. The transform is based on the Laguerre polynomials, associated Laguerre functions, and their convolution properties.

Part 1 of this paper deals with functions having support only on $[0,\infty)$. The resulting unilateral Laguerre transform finds applications in convolution of such functions, inversion of Laplace transform, and in solution to renewal and related Volterra integral equations.

Part 2 of this paper deals with functions having support on $(-\infty,\infty)$ via a bilateral Laguerre transform which is an extension of the unilateral transform. Applications of this technique include convolution of such functions and analysis of the Lindley process.

Part 1 has been published in Applied Mathematics and Computation and part 2 has been submitted for publication in that journal.

SUMMARY - PART 1

One often encounters in applied studies integral equations [16] either of form

$$\int_0^x a(x - x')f(x')dx' = b(x) \tag{1}$$

or of the form

$$f(x) - \int_0^x a(x - x')f(x')dx' = b(x) \tag{2}$$

where $a(x)$ and $b(x)$ are specified functions and $f(x)$ is to be found. Equations (1) and (2) are said to be Volterra integral equations of convolution type of the first and second kind respectively. The Neumann series solution of (2) has the form [19]

$$f(x) = b(x) + b(x) * \sum_1^\infty a^{(k)}(x) \tag{3}$$

where the asterisk denotes convolution and $a^{(k)}(x)$ is the k-fold convolution of $a(x)$ with itself.

The entity $\sum_0^\infty a^{(k)}(x)$ and matrix variants associated with systems of integral equations of convolution type arise in operations research [ ], engineering [ ], and biological studies [10].

Sometimes, differential-integral equations give rise to expressions such as

$$s(\tau) = \sum_{k=0}^{\infty} e^{-\lambda\tau} \frac{(\lambda\tau)^k}{(k+1)!} a^{(k+1)}(\tau) \qquad (4)$$

which describes the busy-period density for certain M|G|1 queueing systems [18].

In easy cases the integral equations may be solved analytically via Laplace transformation, and full answers may be obtained when the Laplace transforms are invertible. More often than not, such transforms cannot be inverted and expressions such as (4) are of limited value when they cannot be evaluated explictly. The Laguerre transformation techniques developed in this paper may then be of value.

The deconvolution problem of finding f(x) from (1) when a(x) and b(x) are known numerically, say, is particularly troublesome, and start-up difficulties described below may make conventional numerical procedures useless.

The Laguerre transform techniques described map continuum functions into sequences, and map the continuum convolution operation into lattice convolution of these sequences. Such discrete convolutions are well matched to modern computer competence, and the inversion mapping back to the continuum is direct.

Laguerre transformation has been developed as a tool for the solution of differential equations [12]. The applications of interest here are quite different and new tools have been needed to convert the underlying simple idea into a flexible working procedure adapted to computer requirements.

The first section introduces the Laguerre transform $T: f(\tau) \to (f_n^{L+})_0^\infty$ in a form convenient for our needs. One has

$$f(\tau) = \sum_{n=0}^{\infty} f_n^{L+} \ell_n(\tau) \tag{5}$$

for any square-integrable function $f(\tau)$ on $(0,\infty)$, where $\ell_n(\tau) = L_n(\tau)e^{-\tau/2}$ are the classical orthonormal Laguerre functions and $L_n(\tau)$ are the Laguerre polynomials. The notation of Abramowitz and Stegun [1] is employed throughout. Orthonormality provides the inverse transformation

$$f_n^{L+} = \int_0^\infty f(\tau)\ell_n(\tau)d\tau \ . \tag{6}$$

Let $T_f^{L+}(u) = \sum_0^\infty f_n^{L+} u^n$ be the generating function of f_n^{L+}. Then one has, as shown in Section 1,

$$T_f^{L+}(u) = \frac{1}{1-u} \ c\left(\frac{1}{2} \ \frac{1+u}{1-u}\right) \tag{7}$$

where $\phi(s)$ is the Laplace transform of $f(\tau)$. This relationship permits evaluation of $f_n^{L\dagger}$ for many important $f(\tau)$.

Section 2 provides simple examples of the transform, and Section 3 discusses the structure of $T_f^{L\dagger}(u)$ in the complex u-plane. Such insight into structure in the complex plane is crucial to many of our algorithms and theorems.

Algorithms for the calculation of the Laguerre coefficients are presented in Section 7. Section 8 is devoted to a discussion of the deconvolution problem.

A variety of numerical examples of the method are treated in Section 9, and the implementation of the procedure is discussed.

Section 10 describes interpolation methods and problems when the known functions are known only numerically.

A final section deals with possible generalizations of the method to special families of functions.

SUMMARY - PART 2

In a previous paper, hereafter designated by [A], a description was given of a Laguerre transformation which maps a function $f(\tau)$ in $L_2(0,\infty)$ into a sequence $(f_n^{\#})_0^{\infty}$ on the nonnegative integers. Moreover, for two such functions $f(\tau)$, $g(\tau)$, the convolution $f(\tau) \cdot g(\tau)$ is mapped into the lattice convolution $(f*g)_n^{\#} = \sum_0^n f_{n-m}^{\#} g_m^{\#} = ((f_m^{\#})*(g_m^{\#}))_n$. One obtains thereby an algorithmic basis for the computation of multiple convolutions $f^{(k)}(\tau)$ and related infinite series of importance to statistics and applied probability.

Such Laguerre transforms have one-sided functions as their natural domain because the Laguerre polynomials $L_n(\tau)$ and Laguerre functions $\ell_n(\tau) = L_n(\tau)e^{-\tau/2}$ are associated with the one-sided weight function $e^{-\tau}$ on $(0,\infty)$. Nevertheless, the methods have a simple extension to two-sided functions on the full continuum $(-\infty, \infty)$ via the same Laguerre functions as we will see.

A variety of applications exist to statistics, operations research, and engineering. In statistics, for example, one has need for multiple convolutions of two-sided distributions unavailable analytically, that of the logistic distribution, for example. Even relatively innocuous distributions such as the Laplace distribution convolve with difficulty.

In operations research studies dealing with queues, inventories and storage systems, one encounters as a structural entity [3] the extended renewal density $h(x) = \sum_1^{\infty} a^{(k)}(x)$, where $a(x)$ is a probability density function with two-sided support. For many densities of interest, evaluation of $h(x)$ has been resistant.

In the earlier paper [A], the crucial role of the complex plane in the formulation of the algorithms was evident, even though the algorithms were entirely in the real domain. For the bilateral transform, the complex plane is again very much present, with Laurent expansions, bilateral Laplace transformation and conformal mapping entering as crucial tools.

The first section extends the earlier formalism to the full continuum. That this extension is natural, and not just an artificial piecing together of the formalism for each half-line, will be clear from (1.9), (1.12) and (1.13). The harmony of the basis will also emerge vividly in Section 3, which deals with the extent of the transform coefficients, and associated uncertainty relations. The topic of extent is crucial to the utility of the Laguerre transform method as a numerical tool. Numerical examples are presented in Section 5. A table of contents provides the reader with an overview of the paper.*

*Two references (V. I. Krylov and N. S. Skoblya [8], and H. T. Weeks [12]) have come to the authors' attention subsequent to publication of [A]. Both deal with the use of Laguerre functions for the numerical inversion of one sided Laplace transforms.

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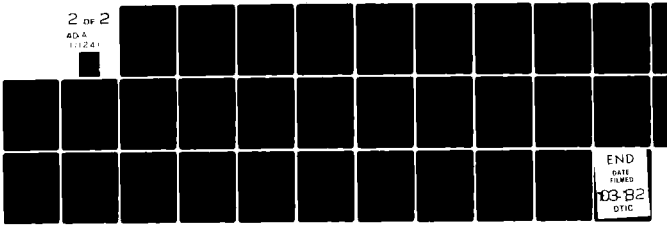

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"Spline Smoothing and Optimal Rates of Convergence in Nonparametric Regression Models"

ABSTRACT

1. The model. Consider the nonparametric regression model

$$y_i = f(x_i) + \epsilon_i \quad , \quad i = 1,\ldots,n,$$

where observations are taken at distinct points assumed for simplicity to be in $[0,1]$. The usual assumptions on the random errors are in force, i.e., $E\,\epsilon_i = 0$, $E\,\epsilon_i \epsilon_j = \delta_{ij}\sigma^2$, but the response function f is assumed only to be sufficiently smooth so that $\|f^{(k)}\|^2 = \int_0^1 f^{(k)}(x)^2\,dx$ exists and is finite.

This model is motivated by certain robustness considerations. For small $\alpha > 0$, the class $\{f$: f has k-1 abs. cont. derivatives, $\|f^{(k)}\| \le \alpha\}$ can be viewed as a collection of response functions at least locally well approximated by polynomials of degree k-1 (or order k). If a regression method is uniformly good in this class, it is robust to arbitrary small departures from the standard kth order polynomial model (see [7]). This concept is also related to the models of Sacks and Ylvisaker [6].

2. A basis for splines and the proposed estimator. A variant of spline smoothing is proposed which is most easily

expressed using the basis of Demmler and Reinsch [2]. Given a distinct knot set $\{x_1,\ldots,x_n\}$, let $\mathcal{S}_n^{2k}$ denote the n dimensional space of natural polynomial splines of degree 2k-1 with simple knots at the prescribed points. There is a basis $\{\varphi_1,\ldots,\varphi_n\}$ for $\mathcal{S}_n^{2k}$ along with eigenvalues $\{\lambda_i\}_{i=1}^n$ determined (essentially uniquely) by the conditions

$$\sum_{\ell=1}^{n} \varphi_i(x_\ell)\varphi_j(x_\ell) = \delta_{ij}$$

$$i,j = 1,\ldots,n.$$

$$\int_0^1 \varphi_i^{(m)}(x)\varphi_j^{(m)}(x)dx = \delta_{ij}\lambda_j$$

Let $\hat{\beta}_j = \sum_{i=1}^{n} y_i\varphi_j(x_i)$. (Note that $\hat{\beta}_j$ is the least squares estimator of β_j if $f = \sum_{j=1}^{n} \beta_j\varphi_j \in \mathcal{S}_n^{2k}$.) With this basis, the family of estimators to be studied, indexed by a parameter $\lambda \geq 0$, has the form

$$\hat{f}_\lambda(x) = \sum_{i=1}^{n} (1-\sqrt{\lambda\lambda_i})_+\beta_i\varphi_i(x).$$

One can see that $\hat{f}_\lambda$ is a natural polynomial spline, but it is <u>not</u> the smoothing spline of Reinsch [5].

<u>3. Optimal rates of convergence</u>. The purpose of introducing $\hat{f}_\lambda$ is that it is minimax in the following sense. Let $\mathcal{C}$ be the class of all estimators $\hat{f}$ which are linear in the observations. (Clearly $\hat{f}_\lambda \in \mathcal{C}$.) For fixed f and $\hat{f} \in \mathcal{C}$ define $T(f,\hat{f}) = \frac{1}{n}\sum_{i=1}^{n} (f(x_i)-\hat{f}(x_i))^2$. Then the next result follows from Kuks and Olman [4].

THEOREM 1. $$\min_{\hat{f}\in\mathcal{C}} \max_{\|f^{(k)}\|\le\alpha} ET(f,\hat{f}) = \min_{\lambda} \max_{\|f^{(k)}\|\le\alpha} ET(f,\hat{f}_\lambda)$$

$$= \min_{\lambda} \{\alpha^2\lambda + \sigma^2 \sum_{i=1}^{n} (1-\sqrt{\lambda\lambda_i})_+^2\} .$$

From the last expression, the minimax rate of convergence of $ET(f,\hat{f}_\lambda)$ is seen to depend critically on the eigenvalues $\{\lambda_1,\ldots,\lambda_n\}$. In this paper an approximation is given when the x_i's are equally spaced, although similar results undoubtedly hold for any setting where the points are suitably regular. The key tool is an estimate first given by Utreras [7] and due in the form here to the author.

LEMMA. For $k \ge 2$, there exist constants d_k such that $\lambda_j = d_k \frac{j^{2k}}{n}(1+o(1))$ uniformly in j for $j = 0(n^{2/(2k+1)})$. With this estimate the best possible uniform rate of convergence follows.

THEOREM 2. For equally spaced points and $k \ge 2$,

$$\min_{\hat{f}\in\mathcal{C}} \max_{\|f^{(k)}\|\le\alpha} ET(f,\hat{f}) = n^{-2k/(2k+1)} c_k(\alpha,\sigma)(1+o(1))$$

where $c_k(\alpha,\sigma)$ is a constant depending only on k, α, and σ.

This rate compares with recent work of Stone [8]. There the same rate is obtained under weaker conditions but without the exact constant.

In the usual situation there is a single fixed but unknown f. Let λ^* be the minimizer of $ET(f,\hat{f}_\lambda)$.

LEMMA. $ET(f,\hat{f}_{\lambda^*}) = n^{-2k/(2k+1)}\, c_k(\|f^{(k)}\|,\sigma)(1+o(1))$ and $\lambda^* = O(n^{1/(2k+1)})$.

4. An adaptive estimator. Since in practice neither $\|f^{(k)}\|$ nor σ is generally known, an experimentor is forced to choose, subjectively or otherwise, some value of the smoothing parameter λ to use. This problem is characteristic of virtually every nonparametric regression method. The procedure here is to estimate λ by Wahba's method of generalized cross validation. The main new result is weak consistency and asymptotic optimality of the resulting adaptive method. This procedure is closely related to the one in Craven and Wahba [1] where ordinary smoothing splines are used. However the consistency result here is much stronger.

Since $\hat{f}_\lambda$ is a linear estimate, there is an $n \times n$ matrix $A(\lambda)$ such that $\hat{f}_\lambda \equiv (\hat{f}_\lambda(x_1),\ldots,\hat{f}_\lambda(x_n))' = A(\lambda)y$. Then the cross validation function is defined to be

$$V_n(\lambda) = \frac{1}{n}\|y-\hat{f}_\lambda\|^2/[\tfrac{1}{n}\mathrm{Tr}\,(I-A(\lambda))]^2 .$$

Also, redefine

$$T_n(\lambda) = \frac{1}{n}\sum_{i=1}^{n}(f(x_i)-\hat{f}_\lambda(x_i))^2 = T(f,\hat{f}_\lambda).$$

Then a straight forward application of the GCV theorem of Golub, Heath, and Wahba [3] yields

$$\frac{EV_n(\lambda)-\sigma^2-ET_n(\lambda)}{ET_n(\lambda)} = o(1)$$

uniformly in $0 \le \lambda \le \lambda_n^o$, where λ_n^o is some sequence satisfying $\lambda_n^*/\lambda_n^o \to 0$. This implies that the minimizers of $EV_n(\lambda)$ and $ET_n(\lambda)$ asymptotically coincide. If in addition normality is assumed, the following much stronger result is true.

LEMMA. Suppose the distribution of y_i is normal for $i = 1,\ldots,n$. Then there is an estimator $\hat{\sigma}_n^2$ of σ^2 such that

$$\frac{V_n(\lambda)-\hat{\sigma}_n^2 - ET_n(\lambda)}{ET_n(\lambda)} = o_p(1)$$

uniformly in $0 \le \lambda \le \lambda_n^o$. From this the main result follows.

THEOREM 3. Let λ_n^* be the minimizer of $ET_n(\lambda)$. Under the assumptions above, there is a sequence of (possibly local) minimizers $\{\hat{\lambda}_n\}$ of $V_n(\lambda)$ such that $\hat{\lambda}_n/\lambda_n^* \xrightarrow{P} 1$. Moreover, $T_n(\hat{\lambda}_n)/ET_n(\lambda_n^*) \to 1$.

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"Incomplete Gamma Functions, Numerical and Asymptotical Aspects for Evaluation and Inversion"

ABSTRACT

The following topics will be discussed:

- The asymptotic expansion of integrals of the type

$$F_a(x) = (a/\pi)^{\frac{1}{2}} \int_x^\infty e^{-at^2} f(t)\, dt, \quad a \to \infty \tag{1}$$

which is uniformly valid with respect to x in a domain that contains $x = 0$.

- The inversion (for large a) of $F_a(x)$; i.e., the computation of x from the equation $F_a(x) = q$, where q and a are given (a large).
- The numerical evaluation of the incomplete gamma functions for large values of their parameters. The algorithm is based on the asymptotic representation of $F_a(x)$.
- The inversion of the incomplete gamma functions for large values of their parameters.

We suppose that in (1) f is analytic in a domain of the complex t-plane that contains the real axis, and that $f(t) > 0$ for real (finite) t. Furthermore we suppose that $F_a(-\infty) = 1$. Several distribution functions of mathematical statistics can be written as (1), for instance the incomplete gamma functions. The function f may depend on a; we are allowing a representation

$$f(t) \sim \sum_{n=0}^{\infty} \frac{f_n(t)}{a^n}, \qquad a \to \infty,$$

in which f_n and f have the same domain of analyticity.

The asymptotic expansion of $F_a(x)$ is obtained by an integration by parts procedure. It has the form

$$(2) \qquad F_a(x) \sim \tfrac{1}{2} \operatorname{erfc}(xa^{\frac{1}{2}}) \sum_{n=0}^{\infty} \frac{A_n}{a^n} + e^{-ax^2} \sum_{n=0}^{\infty} \frac{B_n(x)}{a^{n+1}}.$$

The A_n and B_n are defined recursively in terms of f. They have the same domain of analyticity as f.

The inversion of $F_a(x) = q$, $0 < q < 1$, is for large a based on (2). First we solve by using known algorithms the equation

$$\tfrac{1}{2} \operatorname{erfc}(xa^{\frac{1}{2}}) = q,$$

giving for x an approximation x_0. Then an asymptotic expansion

$$x \sim x_0 + \frac{1}{a}x_1 + \frac{1}{a^2}x_2 + \ldots$$

is derived, in which the x_n are expressed in terms of f, for instance

$$x_1 = \frac{\ln(1 + f(x_0) - f(0))}{x_0}.$$

The expansion is especially useful in the neighborhood of $x = 0$, i.e., for values of q near $\frac{1}{2}$. Numerical experiments for the case of the incomplete gamma functions show uniformity of (3) with respect to $q \in [0,1]$. For these functions more information on x_n will be given.

In general, the expansion in (2) is too complicated for numerical computations. This will be illustrated for the incomplete gamma functions (see also [1]). An alternative is proposed, which in part is based on (2), and which gives an efficient and reliable algorithm for the computation of

$$(4) \qquad P(a,x) = \frac{1}{\Gamma(a)} \int_0^x e^{-t} t^{a-1}\, dt, \quad Q(a,x) = \frac{1}{\Gamma(a)} \int_x^{\infty} e^{-t} t^{a-1}\, dt,$$

for a and x large near the critical line $x = a$. The algorithm runs as follows. Introduce for $x \geq 0$ and $a > 0$

(5) $\lambda = x/a, \quad \eta = \{2[\lambda-1-\ln \lambda]\}^{\frac{1}{2}}$

with sign η = sign$(\lambda-1)$ $(\lambda > 0)$. Then (see [1])

(6)
$$P(a,x) = \tfrac{1}{2}\,\mathrm{erfc}[-(a/2)^{\frac{1}{2}}\eta] - R_a(\eta)$$
$$Q(a,x) = \tfrac{1}{2}\,\mathrm{erfc}[\,(a/2)^{\frac{1}{2}}\eta] + R_a(\eta)$$

and $R_a(\eta)$ has an expansion as the second series in (2). The function $S_a(\eta)$ in

$$R_a(\eta) = (a/2\pi)^{\frac{1}{2}}\, e^{-\frac{1}{2}a\eta^2}\, S_a(\eta)$$

is for $a > 0$ and $\eta \in \mathbb{R}$ slowly varying and it is analytic in a neighborhood of $\eta = 0$:

(7) $$S_a(\eta) = \sum_{k=0}^{\infty} s_k(a)\eta^k \qquad |\eta| < 2\pi^{\frac{1}{2}}.$$

It satisfies the differential equation

$$\frac{d}{d\eta} S_a(\eta) + a\, S_a(\eta) = 1 - \frac{1}{\Gamma^*(a)} \frac{\eta}{\lambda-1}$$

where $\Gamma^*(a) = (a/2\pi)^{\frac{1}{2}}\, e^a a^{-a}\Gamma(a)$ $(a > 0)$, the relation between η and λ being given in (5). The s_k in (7) are easily obtained from a recursion relation, which for numerical applications is used in backward direction. Instead of (7) we can expand $S_a(\eta)$ in Chebyshev polynomials $T_k(\eta/\eta_0)$, for some $\eta_0 > 0$, yielding an even beter expansion. In both cases we obtain expansions, which converge faster as a increases, and from which $S_a(\eta)$ can be computed for, say, $-1 < \eta < 1$, or equivalently for $0.3017.. < \lambda < 2.357 \ldots$. This gives an algorithm for the functions in (4) for

$$0.3017a < x < 2.357a.$$

Initially we supposed large values of a. The algorithm works quite well for $a \geq 5$.

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"Applications of Spheroidal Wave Functions to Time Series Analysis"

ABSTRACT

Given a finite sample of a process some of the major problems of time series analysis are those of testing for the presence of a line component, choosing an algorithm to estimate the spectrum so that the estimate is not dominated by bias, ensuring that the estimate is consistent and statistically meaningful, and maintaining these properties in the presence of minor variations of assumptions. Despite a long history these problems are still lacking satisfactory solutions in all but the simplest cases.

We assume a finite sample $\{x_0, x_1, \cdots, x_{N-1}\}$ of a wide sense stationary time series having the centered Cramér representation

$$x_n = \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{i2\pi\nu\left(n-\frac{N-1}{2}\right)}\, dZ(\nu)$$

The extended Cramér representation permits a distinction between harmonic analysis and spectrum estimation: *harmonic analysis* is concerned with the *first* moments of $dZ(\nu)$, while *spectrum analysis* is the problem of estimating the *second* moments of $dZ(\nu)$.

These moments are estimated as functions of the discrete Fourier transform of the observations which, for notational simplicity, it is convenient to define in centered form:

$$\tilde{x}(f) = \sum_{n=0}^{N-1} e^{-i2\pi f\left(n-\frac{N-1}{2}\right)} x_n$$

Using the spectral representation for the data in this formula we have

$$\tilde{x}(f) = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\sin N\pi(f-\nu)}{\sin \pi(f-\nu)}\, dZ(\nu) \qquad (1)$$

which is the convolution of the Cramér process, $dZ(\nu)$, with a Dirichlet kernel.

In these paper we give a new solution to these problems obtained by applying a "localized" Karhunen-Loève, or principle components, expansion *in the frequency domain* to estimate the moments of the Cramér process, $dZ(\nu)$. From this viewpoint equation (1) is best regarded as a linear Fredholm integral equation of the first kind for $dZ(\nu)$ and, since detailed information about the eigenfunctions and eigenvalues of the Dirichlet kernel have recently been published by Slepian [1978], it is feasible to attempt it's solution. These eigenfunctions, denoted by $U_k(N, W; f)$, $k = 0, 1, \cdots N-1$ are known as *discrete prolate spheroidal wave functions* and are solutions of the equation:

$$\int_{-W}^{W} \frac{\sin N\pi(f-f')}{\sin \pi(f-f')} U_k(N, W; f')\, df' = \lambda_k(N, W) \cdot U_k(N, W; f)$$

Because the Dirichlet kernel is degenerate it is impossible to obtain exact or unique solutions; what we attempt is an approximate solution which is both numerically and statistically plausible.

Within this framework the harmonic analysis line test procedure becomes essentially an analysis of variance applied to the coefficients of the eigenexpansion and so results in an approximate likelihood test. Similarily, from the spectrum estimation viewpoint, the technique used to approximately solve the fundamental integral equation results in an estimate which is data adaptive and computationally equivalent to using the weighted average of a *series* of direct spectrum estimates made with *orthogonal* data windows (discrete prolate spheroidal sequences) applied in the time domain. Since the expansion is applied in the frequency domain it is insensitive to minor departures from normality and the time domain aspects of the procedure permit it to be easily robustified against gross outliers.

While this procedure is philosophically very different from the various autoregressive and maximum-entropy methods currently fashionable the analysis of variance procedure provides considerable insight into the "super-resolution" question. Further, in addition to providing estimates of the spectrum which are based on well-established principles instead of heuristics this methodology also permits a resolution of the differences between windowed and unwindowed philosophies.

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"Absolute Error Bounds for Edgeworth Asymptotic Expansions"

ABSTRACT

1. Introduction

In this paper, we propose the use of MACSYMA to aid in obtaining absolute error bounds for a class of asymptotic expansions. The uniform asymptotic expansions of interest here are those referred to in statistical literature as Edgeworth series. The Edgeworth series can produce quickly converging and accurate approximations to statistical distribution functions whose computations are ordinarily intractable. This is especially true for noncentral distributions with large parameters which include the noncentral beta and gamma distributions. Unfortunately, the only way to determine the accuracy of these asymptotic expansions is by comparison to known values.

The plan of attack in determining error bounds is to use MACSYMA to find a continued fraction 'corresponding' to the asymptotic series. Bounding an asymptotic series or summing a divergent series via continued fractions is certainly not new (Wall [10], Henrici [6], Shenton and Bowman [9]). However, this will be the first time the technique has been applied to the rather complex Edgeworth series. This fact is not surprising because the proposed technique would be more than formidable without MACSYMA.

2. The Edgeworth Series

Various derivations of the Edgeworth series can be found in the statistical literature. See Berry [1], Draper and Tierney [3], Esseen [4], and Hsu [8]. We prefer an exposition similar to that of Hill and Davis [7], because the series is presented in an explicit, easily programmable form, which eliminates the bother and possible mistakes in the production and use of numerous tabled constants. See Draper and Tierney [3].

Given a random variable, X, with pdf, $f(x|\underset{\sim}{\theta})$; where $\underset{\sim}{\theta}$ is a possibly vector parameter. The characteristic function (cf) (or Fourier-Stieltjes transform) of $f(x|\underset{\sim}{\theta})$ is denoted:

$$\phi_X(t|\underset{\sim}{\theta}) = E[\exp(itx)] = \int_{-\infty}^{\infty}\exp(itx)f(x|\underset{\sim}{\theta})dx. \qquad (2.1)$$

If a power series expansion in (it) exists for $\ln \phi_X(t|\underset{\sim}{\theta})$, i.e.,

$$\ln \phi_X(t|\underset{\sim}{\theta}) = \sum_{j=1}^{\infty} K_X(j)(it)^j/j!, \qquad (2.2)$$

which is usually valid only if $|t| < 1$, then the coefficients, $K_X(j)$, of $(it)^j/j!$ are the "cumulants" of the distribution of the random variable, X.

This starting point for developing the Edgeworth expansion is closely related to the central limit theorem, in that we can view the random variable, X, as being the sum of $n = \sigma^2 = K_X(2)$ independently and identically distributed (iid) random variables. The cf of the sum of n iid random variables has the form $[\phi(t)]^n$. Clearly, if the cumulant expansion (2.2) exists, $\phi_X(t|\underset{\sim}{\theta}) = [\exp\{(\ln\phi_X(t|\underset{\sim}{\theta}))/\sigma^2\}]^{\sigma^2}$ which implies that the random variable, X, can be viewed as the sum of $n = \sigma^2$ iid random variables with cumulants $K_X(j)/\sigma^2$.

If we make the transformation, $Y = (X-\mu)/\sigma$, and note that the cumulants of the standardized random variable, Y, are:

$$\begin{aligned} K_Y(1) &= K_X(1) - \mu = 0, \\ K_Y(2) &= K_X(2)/\sigma^2 = 1, \text{ and} \\ K_Y(j) &= K_X(j)/\sigma^j, \text{ for } j \geq 3; \end{aligned} \qquad (2.3)$$

we can write the cf of Y in terms of the cumulants of X:

$$\begin{aligned} \phi_Y(t|\underset{\sim}{\theta}) &= \exp\left\{-t^2/2 + \sum_{j=3}^{\infty} K_X(j)(it/\sigma)^j j!\right\} \\ &= \phi_Z(t) \cdot \exp\left\{\sum_{j=1}^{\infty} \lambda(j)(it)^{j+2}/(\sigma^j(j+2)!)\right\}; \end{aligned} \qquad (2.4)$$

where $\lambda(j) = K_X(j+2)/\sigma^2$, and $\phi_Z(t) = e^{-t^2/2}$, the cf of a standard normal random variable. Cearly, if $\lim_{\sigma\to\infty}\lambda(j)/\sigma^j = 0$, then $\lim_{\sigma\to\infty}\phi_Y(t|\underset{\sim}{\theta}) = \phi_Z(t)$, or the limiting distribution of Y is standard normal. This is the case for each of the four distributions we have investigated.

Expanding the exponential of (2.4) in a Taylor series, we have:

$$\phi_Y(t|\underset{\sim}{\theta}) = \phi_Z(t)\left\{1 + \sum_{k=1}^{\infty}\left[\sum_{j=1}^{\infty}\lambda(j)(it)^{j+2}/(\sigma^j(j+2)!)\right]^k/k!\right\}; \quad (2.5)$$

which is valid for $|t| < 1$. The fact that the series (2.5) is not valid for all t was the source of some confusion regarding the convergence properties of the Edgeworth series (see Fisher [5]). Cramer [2] ended the claims that the Edgeworth series is convergent.

Now, if we expand (2.5), collecting terms with the same power of σ, we have a polynomial in (σ^{-1}) whose coefficients are polynomials in powers of (it):

$$\phi_Y(t|\underset{\sim}{\theta}) = \phi_Z(t)\left\{1 + \sum_{j=1}^{\infty}B_j(it)\sigma^{-j}\right\}; \quad (2.6)$$

where $B_1(it) = \lambda_1(it)^3/3!$,

$B_2(it) = \lambda_2(it)^4/4! + \lambda_1^2(it)^6/((3!)^2 2!)$, and

$B_3(it) = \lambda_3(it)^5/5! + \lambda_1\lambda_2(it)^7(3!4!) + \lambda_1^3(it)^9/(3!)^4$,

etc.

With a minor modification of the notation used by Hill and Davis [7], we can develop a general expression for $B_j(it)$ as follows.

Denote by Π_j, a partition of the positive integer, j, into ℓ positive integers:

$$\Pi_j = \left[s_1^{\rho_1}, \ldots, s_k^{\rho_k}\right], \; j = \sum_{i=1}^{k}\rho_i s_i, \; \ell = \sum_{i=1}^{k}\rho_i, \quad (2.7)$$

and define three functions of the partition, Π_j:

$$m(\Pi_j) = \sum_{i=1}^{k}\rho_i(s_i+2) = 2\ell+j, \quad (2.8)$$

$$a(\Pi_j) = \left[\prod_{i=1}^{k}\rho_i!((s_i+2)!)^{\rho_i}\right]^{-1}, \text{ and} \quad (2.9)$$

$$\lambda(\Pi_j) = \prod_{i=1}^{k} \sigma_{s_i}^{\rho_i}. \tag{2.10}$$

Combining equations (2.8), (2.9), and (2.10), we have:

$$B_j(it) = \sum_{\Pi_j} a(\Pi_j)\lambda(\Pi_j)(it)^{2\ell+j}; \tag{2.11}$$

where the summation is taken over all distinct partitions of the integer, j.

The next step is to invert the cf (2.6) to cover the pdf of the random variable, Y. The inversion formula is:

$$f_Y(y|\underset{\sim}{\theta}) = (2\pi)^{-1}\int_{-\infty}^{\infty}\exp\{-ity\}\phi_Y(t|\underset{\sim}{\theta})dt. \tag{2.12}$$

Applying (2.12) to (2.6), which is valid only for $|t| < 1$, produces the Edgeworth series for the pdf. Note that the result does <u>not</u> equal $f_Y(y|\underset{\sim}{\theta})$ because the range of integration exceeds the radius of convergence of the expansion.

To apply (2.12) to the expansion (2.6) term by term, the following two results are required:

a) From Fourier analysis,

$$(-1)^{j-1}\phi^{(j)}(y) = (2\pi)^{-\frac{1}{2}}\int_{-\infty}^{\infty}(it)^{j-1}\exp\{-ity-t^2)/2\}dt; \tag{2.13}$$

where $\phi^{(j)}(y)$ = the jth derivative of a standard normal cdf. (Note: $\phi^{(1)}(y) = f_Z(y)$.)

b) From Rodrigues' Formula for Hermite Polynomials,

$$\phi^{(1)}He(y|j-1) = (-1)^{j-1}\phi^{(j)}(y). \tag{2.14}$$

Applying (2.13) and (2.14) to (2.6), the powers of (it) in (2.11) can be immediately replaced by the corresponding Hermite polynomial, yielding the Edgeworth asymptotic expansion for the pdf of the random variable, Y:

$$f_Y(y|\underset{\sim}{\theta}) \simeq \hat{f}_Y(y|\underset{\sim}{\theta},n) = \phi^{(1)}(y)\{1 + \sum_{j=1}^{n} C_j(y)\sigma^{-j}\}; \text{ where} \tag{2.15}$$

$$C_j(y) = \sum_{\Pi_j} a(\Pi_j)\lambda(\Pi_j)He(y|2\ell+j). \tag{2.16}$$

Finally, to get the cdf $\left[F_Y(y|\underset{\sim}{\theta}) = \int_{-\infty}^{y} f_Y(u|\underset{\sim}{\theta})du\right]$, the integration is performed term wise to yield:

$$F_Y(y|\underset{\sim}{\theta}) \simeq \hat{F}_Y(y|\underset{\sim}{\theta},n) = \phi^{(0)}(y) - \phi^{(1)}(y)\left\{\sum_{j=1}^{n} D_j(y)\sigma^{-j}\right\}; \quad (2.17)$$

where $D_j(y)$ are the same as $C_j(y)$ except each Hermite polynomial is reduced in order by 1, i.e.,

$$D_j(y) = \sum_{\Pi_j} a(\Pi_j)\lambda(\Pi_j)He(y|2\ell+j-1). \quad (2.18)$$

3. Determining a J-fraction Corresponding to the Edgeworth Series

To generalize the Edgeworth series, let $f(y|\underset{\sim}{\theta},\sigma)$ be a function of y, depending on a possible vector parameter, $\underset{\sim}{\theta}$ and σ. Then the Edgeworth expansion of the function is $f(y|\underset{\sim}{\theta},\sigma) \simeq \sum_{j=0}^{\infty} P_j(y|\underset{\sim}{\theta})\sigma^{-j}$, with $P_j(y|\underset{\sim}{\theta})$ polynomials. Wall (pp. 362-3) [10], gives several criteria for establishing the existence of a continued fraction, J-fraction in Wall's terminology, corresponding to a given power series. The J-fraction is of the form:

$$\cfrac{a_0}{b_1 + \sigma - \cfrac{a_1}{b_2 + \sigma - \ddots}}$$

If the Edgeworth series had been derived directly from the Fourier-Stieltjes transform, without going through the various transformations and rearrangements, the existence and convergence of the J-fraction could be established in a simpler manner (Henrici [6]). However, for the Edgeworth series, the path of least resistance seems to be to use a quotient-difference type of algorithm (Henrici [6], Wall [10]), to determine if a tractable law for the J-fraction coefficients exists. This is where the powerful functions of MACSYMA can be used; i.e., the algorithms can be implemented symbolically and hopefully the resulting expressions will be reduceable to the point where some law of formation can be discovered. [Note: in the quotient-difference type algorithms, the coefficients of the J-fraction are obtained in terms of the coefficients of the Edgeworth series.]

A starting point would be to try the above approach on one of the simpler Edgeworth expansions, such as that for the incomplete gamma function. Assuming success here (or failure), one could approach the Edgeworth series for central and noncentral beta distributions. If the law for the J-fraction coefficients can be determined through MACSYMA, then the next step is to try to establish convergence of the continued fraction to the desired function. Doing that, the Edgeworth asymptotic expansion can then be bounded through the J-fraction.

4. Conclusions

Perhaps the application of MACSYMA to the problem of bounding Edgeworth asymptotic expansions will not produce any good results. In any case, it can be no less successful than the many previous attempts by other methods. It is hoped that the occasional orderliness of statistical distribution functions will surface in this attempt.

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"On Computer Architectures for Statistical Algorithms"

ABSTRACT

Statisticians have recently been concerned with robust regression concepts. Introduction of M-estimation procedures somewhat extends the computational burden over ordinary multiple linear models with normal errors, but does not yet begin to test the computational capabilities of modern computer architectures. Consider, for contrast, a reasonably straightforward application in image processing. An image may be represented as a function, $f: R^2 \to R$, taking the real plane into the real line, $z = f(x,y)$ being the intensity at point (x,y) and x and y be respectively the horizontal and vertical locations of a point in the image. A very simple noisy picture may be represented as a regression problem by

$$z_{ij} = f(x_i, y_j) + \varepsilon_{ij} \qquad (1)$$

where the ε_{ij} are the usual white Gaussian noise. This problem corresponds to a snowy TV picture in weak reception area. Since f in general is a strongly nonlinear function of x_i and y_j it is clear that some nonlinear nonparametric regression methodology is necessary even in this simple case. Perhaps the closest work to solving this problem is that of Wahba (1979). See also Wegman and Wright (1980). Even this spline approach is not entirely adequate since splines are required to be smooth and an image may have very sharp

discontinuities (high contrast). In certain satellite remote sensing applications, imaging sensors operate at very low light levels where individual photons are counted. The white Gaussian noise assumption is then replaced by a Poisson noise distribution. Moreover since these light levels are in a nonlinear response region for film or other imagers (due to so called reciprocity failure), the additive noise assumption is no longer appropriate. Thus a regression model might take the more complex form

$$z_{ij} = f(x_i, y_i) \circ \varepsilon_{ij} \tag{2}$$

where $\circ$ is a nonlinear binary operator and ε_{ij} (to coin a phrase) is dark non-Gaussian noise. To complicate the picture even more, we could ask for color images which means that f must be a vector valued map, e.g., $f: R^2 \to R^3$. We might model this as

$$(r_{ij}, b_{ij}, g_{ij}) = f(x_i, y_j) \circ \varepsilon_{ij} \tag{3}$$

where r_{ij}, b_{ij} and g_{ij} represent the red, blue and green components of the image. Notice that r_{ij}, b_{ij} and g_{ij} will in general be correlated random variables. One further complication is to suggest we might be interested in a motion picture. We thus introduce a time series aspect to our evolving model which now may look like

$$(r_{ijk}, b_{ijk}, g_{ijk}) = f(x_i, y_i, t_k) \circ \varepsilon_{ijk} \,. \tag{4}$$

and clearly $f: R^3 \to R^3$. To put some simple numbers to this example will help

clarify our point. Even discounting the computational burden required to estimate f in the first place, a high resolution image normally is represented digitally on 1024 x 1024 grid (of pixels). Thus each image requires 2^{20} evaluations of f (itself a vector-valued function). For a simple 20 minute film there are 28,800 images. If we were doing the digital processing image by image, this would mean we had 28,800 nonlinear, non-parametric non-Gaussian regressions to do and to evaluate each of the 28,800 vector-valued function estimators at 2^{20} points. All this is required to process digitally 20 minutes of color film taken in poor light, a relatively realistic assignment. The dimensionality and sample size requirements clearly demonstrate the need for innovative development of statistical algorithms based on a sound knowledge of modern computer architectures.

Some recent developments in microelectronic technology have revolutionized computer design. Very large scale integrated circuit technology (VLSI) has revolutionized the concept of central processing units. VLSI circuit chips now can contain a multiplier which makes parallel and network arrangements of processors possible in a relatively inexpensive fashion. Processors may be connected, for example, not only in parallel arrays but in orthogonal or hexagonally connected arrays. Such innovative computer architectures allow for a totally different approach to algorithm development. Some examples using matrix manipulations suggest that statistical algorithms for multi-dimensional data can be formulated in a fundamentally different way. See, for example, Kung and Leiserson (1978) or Mead and Conway (1980). We advocate

an organic approach to algorithm development. That is, rather than having a theoretician develop a formula which is then translated by a programmer to a computer algorithm, we think there is much to be derived from an integrated approach.

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"Some Applications of Asymptotics in Statistics"

ABSTRACT

In the study of probabilities of large samples one often encounters integrals of the form

$$(1) \qquad I_n = \int_a^b \phi(x)[f(x)]^n dx,$$

where $\phi(x)$ and $f(x)$ are continuous functions defined on the finite or infinite interval $[a,b]$ and $f(x)$ is positive there. Long ago, Laplace made the observation that the major contribution to the integral should come from the neighbourhoods of the points where $f(x)$ attains its greatest value. Furthermore, he showed that if $f(x)$ attains its maximum value only at the point ξ in (a,b) where $f'(\xi) = 0$ and $f''(\xi) < 0$, then as $n \to \infty$

$$(2) \qquad I_n \sim \phi(\xi)[f(\xi)]^{n+\frac{1}{2}} \left\{\frac{-2\pi}{nf''(\xi)}\right\}^{\frac{1}{2}}.$$

This formula is now known as the Laplace approximation.

However, it is not infrequent to come across integrals of the form I_n that do not satisfy the conditions necessary for the validity of the approximation in (2). Hence extensions of and modifications to the method of Laplace must be made in order to obtain the behaviour of these integrals for large values of n. Here we shall consider two such cases. Each of these is illustrated by a specific example.

The first modification concerns the integral

$$I(n) = \int_{-\infty}^{\infty} xe^{-x^2} \left[\frac{1+\theta(x)}{2}\right]^n dx, \tag{3}$$

where $\theta(x)$ is given by

$$\theta(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du. \tag{4}$$

This integral occurred in a problem in probability theory. Note that the function $\frac{1}{2}[1+\theta(x)]$ monotonically increases from 0 to 1 as x varies from $-\infty$ to $+\infty$. Hence the greatest value of this function is not attained at a finite point but at infinity, and the conditions for the Laplace approximation are violated. Nevertheless, we shall show that I(n) has the behaviour

$$I(n) = \frac{\sqrt{\pi \log(n+1)}}{(n+1)} - \frac{\sqrt{\pi}}{4} \frac{\log\log(n+1)}{(n+1)\sqrt{\log(n+1)}} + O\left(\frac{1}{(n+1)\log(n+1)}\right) \tag{5}$$

as $n \to \infty$.

The second modification deals with the incomplete Beta-type integral

$$J(n) = \int_{a_n}^{1} t^{k_n-1} (1-t)^{n-k_n} \, dt, \tag{6}$$

where k_n and a_n are real-valued functions of n satisfying

$$\frac{k_n}{n} = p + O(\tfrac{1}{n}), \tag{7}$$

$$n^k |a_n - a| \to 0 \quad \text{as } n \to \infty \quad \text{for each } k = 1, 2, \ldots, \tag{8}$$

and $0 < p < a < 1$. This integral arises from the study of probabilities of large deviations. If we put

$$f(t) = t^p (1-t)^{1-p} \tag{9}$$

and

$$\phi(t;n) = t^{k_n - np - 1} (1-t)^{np - k_n}, \tag{10}$$

the integral (6) becomes

$$J(n) = \int_{a_n}^{1} \phi(t;n) [f(t)]^n \, dt, \tag{11}$$

which is indeed of the form in (1). Note that the function ϕ and the lower limit a now depend on n and hence the Laplace approximation (2) does not apply directly. However, we shall use a modification of Laplace's method to show that J(n) has the asymptotic expansion

$$J(n) \sim [f(a)]^n \{\frac{c_{1,n}}{n} + \frac{c_{2,n}}{n^2} + \ldots\}, \quad \text{as } n \to \infty, \tag{12}$$

where the coefficients $c_{i,n}$ are bounded functions of n. As an application of this result, we show that the tail probability of the sample p-quantile decays exponentially.

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ATTACHMENT C: List of Participants and Lectures in Global Continuation Methods Portion of Special Year

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List of Lectures

DATE	SPEAKER	TOPIC
Oct. 7, 1980	J. Harrison	Flows without periodic orbits
Oct. 24, 1980	H.-O. Peitgen	Continuation and bifurcation
Feb. 26, 1981	B. C. Eaves	An introduction to solving equations with P.L. homotopies
Mar. 24, 1981	H. Keller	Continuation methods in seismic ray tracing
Mar. 25, 1981	S. Smale	Fundamental theory of algebra: historical remarks and new perspectives from computer science
Mar. 26, 1981	S. Smale	Cost of finding zeros of maps by a variation of Newton's method
Mar. 27, 1981	H. Keller	The optimization problem in continuation and homotopy
Mar. 31, 1981	L. Watson	Homotopy methods--the Colt 45 of nonlinear equation solvers


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